Constructing and sharing mathematical ideas: Some findings from the WebLabs project

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Abstract

This symposium will report the findings of the WebLabs project, a three-year research study funded by the European Union, investigating students' modelling of mathematical and scientific ideas. The fundamental idea of the project is twofold. First, to design and build "transparent modules" (TMs), carefully packaged sets of tools with which students could construct working models of their evolving knowledge in the specific domains chosen. TMs are modules in the sense that each has embedded within it a set of mathematical ideas that are operationalised, that is, the ideas are made to do work by producing something useful or interesting enough to merit further exploration and discussion. They are transparent in the sense that it is relatively straightforward to inspect the mechanism that makes the modules work, to manipulate and change them, and to rebuild them as necessary.

Second, we have designed a "WebReport" system, which serves both as the collaboratively-constructed, public record of the evolving understandings of a knowledge domain among the community, and as the final product of the community's work. The idea is therefore that the WebReports serve as process and product, the way to share working models built with the TMs that would frame the discussion, provide the language by which conflicts could be resolved, and ultimately lead to a co-constructed and consensually validated group report.

The knowledge domains that are the focus of study for our student group – aged between 13 and 15 years are: Infinity, Sequences, Randomness, Fibonacci, 1-D Collisions, Eco-modelling, and Force & Acceleration. In the symposium, each of the partners in the project will describe their approach, and report their findings. The symposium will end with an overview that attempts to draw theoretical and practical lessons from the project.

Exploring Infinite Sequences

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Abstract

This paper describes one of the activities – on the convergence or divergence of infinite number sequences – from the WebLabs project. We built a series of tools in the programming system ToonTalk to generate and investigate infinite number sequences (a topic which is absent from the curriculum for this age-group). One of the advantages of Toontalk is that computational processes and their data are tangible and manipulable (in ToonTalk children enter an animated world full of tools and objects that they can pick up and use in a game-like manner.) Another advantage of this system is that it uses exact arithmetic instead of floating point numbers (used by most other computer environments), allowing for as-precise-as-desired numerical investigations such as that of the convergence of the series 1/2+1/4+1/8+… (with floating point numbers, the sequence of partial sums of the terms of 1/2^n prematurely converges to 1 after 55 iterations). Using this tool, students investigated questions such as whether a sequence can get smaller and smaller and never reach 0.

While the ToonTalk-based toolset proved adequate for generating and observing the sequence as a process, it became clear that it would be useful for students to be able to visualize its characteristics as an object. Adding another representational system would raise, we hoped, structures invisible in the first representation. This would allow them to consider other questions such as the rate-of-change of the sequence and the relationship between the convergence of a sequence and that of its corresponding series. We thus complemented the Toontalk explorations with a graphing tool (Excel) for plotting the data generated in the first representation. In our first trial we found that students were surprisingly competent in switching between the different environments, using each according to the advantages it provides.

Summary

This paper describes one of the activities – on the convergence or divergence of infinite number sequences – from the WebLabs project. These topics are typically considered too complex for children in the early part of secondary school and are excluded from the curriculum, although number sequences are generally studied at a superficial level. Our approach is based on the following principle: that inaccessibility of mathematical ideas are often the result of employing a representational infrastructure that is not well tuned for constructing knowledge, even though it may be perfect for expressing knowledge that an individual or community already has.

From our reading of the relevant literature, it seems that in order to connect intuitions with more formal knowledge, students need to appreciate both process and object views in tandem. Our objective, therefore, has been to devise tools and activities that assist students in coordinating these views, building on what they already know – or what they can 'see' - and to engage with the structure of the sequences and series they are investigating in a quasi-formal way.

The design of our computational tools and activities is based on the following principles:

- Emphasis on the central role of programming as a means of expressing, exploring and formalizing mathematical ideas.
- The use of diverse tools and representations to support the dual view of a sequence as a process and an object.
- The interleaving of construction and argumentation activity – students build and discuss what they have built as seamlessly as we can manage.

A key characteristic of ToonTalk is that every computational process and its data are tangible and manipulable. In addition, the system employs exact arithmetic instead of floating point numbers (used by most other computer environments); this means that numerical investigations, such as those of infinite number sequences and series, can be as precise as desired. This contrasts with the floating-point numbers representation common to most computers where, for example, the partial sums sigma[1-k](1/2^n) prematurely converge to 1 after approximately 55 iterations (when k=55).

We designed a sequence of activities that encouraged students to:

- Uncover and make sense of evidence which challenges their intuitions of infinity.
- Develop a non-algebraic language for describing, discussing and reasoning about sequences, and in particular, the ideas of convergence, divergence and limits.

This activity was tested in London during the spring term of 2004, with a group of 10 students aged 13-14. The experiment included about 8 sessions of 50 minutes each and concluded with a five-hour workshop. At the same time, students in other participating countries undertook similar activities. In the course of the activities, students built a series of tools to generate infinite number sequences and investigate questions and issues such as:

- Can successive elements of a sequence get smaller and smaller and never reach 0?
- What happens when you start from a “very big number” (e.g. 1,000,000), and repeatedly divide it by 3? What about when you divide by 300?
- When you add the terms of a convergent sequence, is the sequence of the partial sums (the series) also convergent?
- Explore the relationship between the rate of convergence of a sequence and the behaviour of the series.

While the ToonTalk-based toolset proved adequate for generating and observing the sequence as a process and investigating these and similar questions from a numerical perspective, it became clear that it would be useful for students to be able to visualize its characteristics in a graphical form. Adding another representational system would raise, we hoped, structures invisible in the first representation, although we are acutely aware that with representations, more is not necessarily better. We thus complemented the ToonTalk explorations with a graphing tool (Excel) for plotting the data generated in the first representation. The tool provided for students to identify a process of a sequence as a ToonTalk stream of numbers, and analyze it as an object by observing it as an Excel graph.

Our design engaged students in activities such as exploring the reciprocals (1/n) and the harmonic series (sigma(1/n)), as an example of a sequence which converges and its series diverges. In a later phase, students focused on the sequence 1/2^n and the series sigma(1/2^n), as an example where they both converge.

In the presentation, we will outline some findings from the activities, based on data from the London site and other collaborating sites. Here, we present two of these findings. The first relates to a change in student's conception of rate-of-change. Their initial intuitions portray all sequences as linear: when asked to predict the shape of the graph for sigma(1/n), students drew a diagonal going up from the origin. Using ToonTalk to construct the sequence and observe it numerically did not seem to change this linear intuition. Yet after the set of sequences was plotted in Excel, students' intuitive predictions regarding the second set were non-linear curves. Furthermore, they were able to support their arguments using statements such as "this one get smaller much faster that that".

A second phenomenon we encountered surprised us. Most students found modelling 1/2^n easy – some of them did it before we even asked them to. On the other hand, modelling 1/n turned out to be more difficult than we expected. We had not expected this difficulty, since structurally they appear similar: In fact, a good case could be made that the standard notation of 1/2^n is more complex than 1/n. When trying to understand students' difficulty, we realised that in terms of the process, generating 1/2^n is much simpler: repeatedly divide by 2. This realization raises questions regarding the complexity of sequences: how can one measure their mathematical, computational, cognitive and pedagogical complexity? How are these correlated? Is the sequence's complexity identical to that of its' generating function? How much of perceived complexity is inherent to the object under study, and how much is contingent on the chosen representational infrastructure?

Modelling 1-D Collisions: Content and process outcomes

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Abstract

Collisions are among the most basic phenomena in which a quantity remains invariant, and conservation theorems play a fundamental role in physics. Identifying invariants is crucial to the modelling process in both mathematics and physics, while application of the conservation laws is fundamental in learning and doing physics. Research on students' understandings of collisions has shown that students tend not to appreciate that they can simply apply conservation principles to the initial and final states of the interacting system, rather than necessarily analyse the micro-interactions involved during the collision. Conservation laws are often presented in a didactic manner with the result that students identify them as meaningless mathematical algorithms to be applied when working out the solution to the outcome of a collision.

We present work carried out in two sites, Nicosia (Cyprus) and London. Both sites were committed to having students complete a common sequence of activities, which focused on model-building in the context of 1-D collisions. Through collaboration, students actively engaged in a combined inquiry and modelling approach involving the creation, testing, refinement, and validation of models in ToonTalk for the different observed classes of collisions, without applying any mathematical formulae. The Cypriot group focused on evaluating learning outcomes in terms of specifically developed pre- and post-tests covering three distinct areas: conceptual understanding, modelling skills and epistemological awareness. The London group gave primary importance to the process of modelling, the mediating role of tools, and students’ opinions expressed through written reports and group discussions. We present the results and highlight the similarities and differences in approach, and finally seek to demonstrate how they can be pulled together in ways that transcend the limitations of each.

Summary

Collisions are among the most basic phenomena in which a quantity remains invariant, and conservation theorems play a fundamental role in physics (de Jong et al, 1999). Identifying invariants is crucial to the modelling process in both mathematics and physics (Hoyles et al, 1999), while application of the conservation laws is fundamental in learning and doing physics (George et al, 2000). Research on students' understandings of collisions have shown that students tend not to appreciate that they can simply apply conservation principles to the initial and final states of the interacting system, rather than necessarily analyse the micro-interactions involved during the collision (Grimellini-Tomasini et al, 1993).

We present work carried out in Nicosia (Cyprus) and London. Both sites were committed to having students complete a common sequence of activities, which focused on model-building in the context of 1-D collisions. However, the London and Cyprus groups had different learning goals and methodologies. Partners in the WebLabs project have differing backgrounds and approaches, and the project seeks to exploit the diverse data collection and evaluation methodologies to present a richer portfolio of results. The Cypriot group focused on evaluating learning outcomes in terms of specifically developed pre- and post-tests covering three distinct areas, while the London group gave primary importance to the process of modelling, and students' opinions expressed through written reports and group discussions. We present the results and highlight the similarities and differences in approach, and finally seek to demonstrate how they can be pulled together in ways that go beyond the limitations of each.

Aims

The Cypriot group had three learning aims: to promote the development of conceptual understanding, of modelling skills, and of epistemological awareness of the role of conservation laws in science as a fundamental principle in analyzing process end states.
The aims of the London group focused on the process of modelling, that is for students to gain understandings of what it means to make and follow up conjectures, to generalise, to specify the limitations of a model; and, simultaneously, to develop understandings of the behaviour of colliding objects. This is not dissimilar to the aims of Cypriot group but did not expect students to appreciate the important role of conservation laws in science: it did, however, expect students to learn about the ideas underlying the principles of conservation and system states.

Research design

In Cyprus, twenty 13-14 year old students participated in two 1.5 hour sessions per week with two researchers at the computer centre of the University of Cyprus. In London, five 13-14 year old students participated in eight 50 minute weekly sessions and a full-day workshop with two researchers at a school computer centre. The two student groups followed a sequence of activities consisting of a number of cycles around different types of collision, each involving prediction, observation of the collision (video), modelling with ToonTalk, model testing, and consideration of its scope of application.

Cypriot outcomes

Students' responses related to the epistemological awareness tests indicates that prior to instruction the majority of students were not able to appreciate the role of the conservation of a specific quantity (e.g. energy, velocity) in various contexts. After instruction, this was only slightly increased. One explanation for this relative failure was that students defined conservation of momentum operationally in the context of 1-D collisions, but did not engage in activities that allowed them to appreciate the importance of conservation laws in several different domains.

Students' responses in pre-tests evaluating conceptual understanding of 1-D collision phenomena indicate that students hold several intuitive ideas (e.g. two carts with equal and opposite velocities will stop after collision). Post-test results indicate that the majority of students identified the correct direction of the carts after collision, and when asked to explain their reasoning made references to conservation of momentum, conservation of velocity, and the models they created in ToonTalk.

The development of modelling skills was evaluated through tests designed on the basis of five aspects defined for modelling skill and statistically significant differences were found.

London outcomes

Students produced three distinct types of model during the activity sequence. As they progressed, students viewed models that could account for more cases of collision phenomena as being better representations of reality. Student models broke down under certain circumstances, giving rise to behaviour that 'looked incorrect'. In these cases students formed hypotheses about 'what would happen in the real-world', compared this to model behaviour, and determined how to reprogram their model to take account of the situation. Some of these model-testing and evaluation interactions occurred collaboratively between students at a given site. There was also evidence that students made reference to the underlying epistemological concepts, for example, the intuition of invariant quantities emerged in some of the student discussions, in the form of swapping velocities or transferring velocity from one cart to the other, both of which imply that total velocity is conserved.

Discussion

The Cypriot results demonstrate that students improved their understanding of several collision phenomena, developed aspects of modeling skills, and a relatively small number of students seemed to appreciate the importance of conservation laws in studying physics. The London results show that students gained understandings of what it means to model, to make conjectures, to generalise, and to validate their models against real-world phenomena.

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Keywords

Exploring randomness via studying, building and sharing models

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Abstract

This research focuses on the idea of introducing pupils to randomness in terms of studying, building and sharing models that represent different aspects of the knowledge domain. The modelling environment we use includes programmable ToonTalk microworlds, LEGO robots, and cardboard toys. The use of tangible models allows students to construct and manipulate random variables with their fingers, thereby embodying highly abstract concepts.

Probability is a young mathematical discipline; randomness, one of its basic concepts, is still under debate. One reason for this ambiguity is that mathematical formalizations of randomness are based either on common sense or on ideas derived from different scientific contexts.

Attempts at formalization include notions like: “unpredictability”, “lawlessness”, “incomputability”, “incompressibility”, “indeterminism”, etc. These are not competing notions. Each one characterises one of the key aspects of randomness. The models built, shared and discussed by students are designed to illuminate these various facets.

Randomness plays a key role in some dynamic systems. Modelling such systems provides motivation for using randomness as a system component and a conceptual tool. We have designed activities, and a set of tools to support them, in which students model dynamic systems, specifically ecological systems. This activity also raises specific questions about computer modelling of dynamic systems.

The modelling activity is complemented by collaborative authoring of a WebLabsPedia: an online glossary of terms related to randomness. This verbalization activity aims to help students transform their intuitions into more articulated concepts.

Here we describe the educational approach and activity sequence that we set up, and provide examples to discuss findings in terms of how the employed technology contributed to reach our educational goals.

Summary

Introduction

Following a socio-cultural perspective, we see artefacts, digital and physical, as embedding knowledge. Our aim is to design and experiment with artefacts, to be used as mediators, that may enhance the processes of learning concepts of randomness and dynamic systems modelling. The artefacts we have designed are all programmable including ToonTalk microworlds, LEGO robots, and cardboard toys. These artefacts allow pupils to study, build and share models that can mediate different aspects of the knowledge domain. The use of tangible models allows pupils to physically construct and manipulate random phenomena, thereby letting them become bodily engaged with phenomena that otherwise are expressed in more abstract forms. Hence, a key challenge for us has been to support children in articulating their experiences, through models and in words, so that they can be communicated to others.

Throughout our research we investigated how the study, construction, and exchange of models in multi-cultural settings contribute to pupils' learning. In particular, how it may provide new means for children to communicate their knowledge, even though they do not fully share a spoken and written language.

This paper describes the sequence of activities that we developed, and how the studying, building and sharing of models contributed to pupils developing understandings of concepts of randomness and its relation to dynamic systems modelling.

Background

Probability is a young mathematical discipline; randomness, one of its basic concepts, is still under debate. There is not a universally accepted definition, and questions like the following are still open: What is randomness? What is a random phenomenon? Given a phenomenon how can we judge if it is random or not?

One reason for this ambiguity is that mathematical formalizations of randomness are based either on common sense or on ideas derived from different scientific contexts. Attempts at formalization include notions like: “unpredictability”, “lawlessness”, “incomputability”, “incompressibility”, “indeterminism”, etc. These are not competing notions, instead each characterises key aspects of randomness. In our research, the models built, shared and discussed by pupils are designed to illuminate these various facets.

Randomness plays a key role in many dynamic systems. Therefore, modelling such systems provides a useful starting point for introducing pupils to randomness as a system component and as a conceptual tool. We have designed activities, and accompanying supporting tools, in which students model dynamic systems with random components, such as ecological systems. The modelling activity is complemented by pupils' collaborative authoring of a WebLabsPedia: an online glossary of terms related to randomness. This verbalization activity aims to help students transform their intuitions and experiences into more articulated concepts.

Research design

We have iteratively designed and implemented construction tools and an activity sequence for classroom usage. Activities were tested in Italy (23 students, age 12) and Sweden (19 students, age 12). Pupils study and build dynamic models of phenomena, and engage in game like activities where they challenge each other to guess the intention and purposes of each other's models and the role played by the random components.

The activity sequence is structured in cycles where phenomenological experiences alternate with verbalization activities and social practices. In particular the starting point of each cycle is an experience based activity, e.g. the study or building of a model, and the end point is the editing/updating of a "randomness encyclopaedia", which is supported through verbalization activities such as class discussions and WebReports. Below we present three main cycles that we set up:

1. Providing pupils with physical experiences concerning randomness. The activities present different situations based on Lego robots that incorporate random elements. The tasks allow pupils to study the phenomenon and discriminate its random components from non-random ones.
2. Pupils are presented with a special random generator (the random garden) in ToonTalk, which allows them to modify a sample space and study the resulting extractions. Given a set of extractions, pupils are asked to build a random garden that would produce qualitatively the same results.
3. Deepening the idea of building models that involve random phenomena. In ToonTalk, and using random garden tools, pupils are allowed to build models representing phenomena from ecological systems, LEGO robots experiments and everyday experiences such as dice throwing.

Data collection and analysis

Our data includes written protocols, models built by pupils, interviews, WebReports and discussion transcripts. The data was analysed to assess: learning about the idea of randomness; learning about the idea of modelling; contribution of our technology (models, modelling tools, collaboration platform) to the educational goals.

In particular, we focused on pupils' capability to deconstruct complex random phenomena, to detach their random elements, and to employ them as means for building or modifying other models involving random processes. Moreover, we investigated children's understandings of the idea of representations and their development of meta-cognitive model building practices.

In order to study how technology contributed to the educational goals, we focused on how it influenced the ongoing processes of studying/building/sharing models, as well as the ongoing social practices and verbalization activities. In particular we searched for references to the employed technology and to cross-site collaboration, to understand how such references contributed to the learning processes.

Results

A central finding related to modelling concerned that of shifting between perspectives such as “builder”, “tester, and “player”. It allowed the pupils to foresee and reason about the consequences of particular tasks’ solutions without having to implement them fully in their models. Moreover, in face-to-face collaboration this provided efficient divisions of labour among the participants. However, this led to different learning trajectories; some oriented towards construction, others towards higher-level analysis. These abilities turned out to have important consequences to the overall goal of learning about randomness. The capability to change standpoints, and to take external standpoints, provided the pupils with insights about the complexity of random phenomena. In particular, it allowed pupils on one hand to individuate the random and non-random components of a complex phenomenon and on the other hand to analyse different phenomena by comparing their random components.

Mathematical Discovery in the Context of Number Sequences

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Abstract

We start from the assumption that students learn best when they actively construct personal understandings of mathematical concepts and relationships. Problem solving is a central learning activity in mathematics education. In the study of number sequences, students are frequently diverted by visible elements and miss the structural properties, which form the core idea in sequences, even if they might be under the surface.

In this study, we sought to understand how students investigate relationships and discover properties in number sequences. To this end, two mathematical tasks on Fibonacci sequences were provided to twenty elementary students with prior experience in using ToonTalk and Webreports. Students were provided not only with data to confirm or reject a conjecture, but also with ideas and representations to gain insights into the different properties of the Fibonacci sequences. In this paper we present results of the work of two students while engaged in these explorations. To identify how the environment facilitated students to model and explore sequences' properties we used mainly field notes, students' models and video records.

From the analysis of the data, we found that the environment played a primary role in engendering problem solving and discovering properties of number sequences. The environment enabled students to construct models for exploring sequences' properties and solve related problems, to share and discuss their models with other fellow-students across other countries and finally to improve their models and solutions. The WebLabs systems prompted and motivated students to explore several possible solutions to a problem by making connections between the different modes of representations, to seek justifications for their conjectures, and to generalize their findings.

Summary

Introduction

In mathematics teaching the potential has been established of the development of a deep understanding of mathematical concepts and processes via active learning, inquiry and problem solving (NCTM, 2000) or through exploration within environments, such as microworlds (Hoyles et al., 2002).

This paper reports findings from a study within WebLabs concerning students' exploration of Fibonacci sequences. The aim was to evaluate how students investigated relationships in Fibonacci sequences, with students building programs to model sequences and explore their properties starting with the property Fk | Fnk, i.e. F2 | F2n, F3|F3n,. They used the WebReports system to share their knowledge – discuss their conjectures, as well as their techniques for testing those.

Here we report on the work of two students (out of a group of 20) who worked on the activities in an informal setting. We collected field notes, video records and students' WebReports to evaluate our aims. Detailed analysis of these data using interpretative techniques indicated how students' ideas were developed at different times in their investigations.

Results

After explaining the property Fk | Fnk,, a researcher asked the students to find ways to "prove" it. Both students started by writing down some terms of the sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 and 144), doing divisions and searching for pattern using paper and pencil or in the microworld. Chris observed that the second term (1) was a divisor of the 4th, 6th and every other second number of the sequence. He noted though that 1 is a divisor of any number of the sequence. Alex agreed but explained that this occurred only with number 1. Alex decided to work on the fourth number of the sequence (3) and he found that 3 was a divisor of 21 (8th) and 144 (12th). Chris continued with the 6th number of the sequence (8) and he found that it was a divisor of 144 (12th). The students soon became convinced that the property was true. They were asked to prove that the property held true for every combination of numbers, where Fk | Fnk. Alex said immediately: "This sounds like a job for a robot"! (ToonTalk program). They then identified the appropriate ToonTalk tools for their needs, and downloaded them from the WebReports system.

Finally, they posted their findings in a WebReport, which received comments from a researcher in Bulgaria, who suggested that it might be better to first train a robot do a specific division (divide the 4th and 6th number by the 2nd one). Our students liked the idea and Alex wrote the first twenty numbers of the sequence in his notebook, while pointing out that the robot should take the 6th, 9th, 12th etc. number of the sequence and divide them with 2. Chris added that the robot could only check whether these numbers were even numbers. Alex liked the idea but he said that would be extremely difficult in terms of robot's programming. He suggested that their robot should multiply the 3rd number by 2, to get the following numbers. Since the remainders of the divisions were 0, the property holds true. Students with assistance then built and packaged the tool and posted it on the web, while writing a WebReport on their work. They were excited in the next meeting when they received another comment this time from a Bulgarian student which suggested that the inclusion of another robot returning "Yes" when the remainder of a division is 0 or "No" when the remainder is not 0, would improve the aesthetic part of the program-model. The two students constructed an additional robot successfully, while in the meantime they debugged it many times, trying to build the best one they could and posted their final solution in a WebReport.

The unexpected continuation of this activity occurred a few days later. While working on the previous investigation, Lena, from Cyprus, observed a "strange" phenomenon: the division of any two consecutive terms of the sequence had always a non zero remainder.

This conjecture became the focus of the next investigation; the researcher added that it would be really tempting to modify Chris' and Alex's existing robot to explore this challenge, while Chris pointed that Fibonacci sequence was full of mysteries; they couldn't bet on how many more properties underlie in the Fibonacci sequence!

Conclusions

The success of this activity need not be measured only by the mathematical content students digested but also by their appropriation of practices of mathematical exploration and investigation. Students modeled mathematical structures, experimented with them, raised and tested conjectures and generalized these into theories. They built upon each others' knowledge by acting on each others models. They identified and sought out the right tools for the task at hand: an act indicating they had a good grasp of both the task and the tools.

The study showed how the learning environment provided students not only with data to confirm or reject a conjecture, but ideas and representations to model and understand mathematical concepts. The students' processes in the activity showed that they were engaging in making conjectures, exploring and modifying solutions at some level, in a microworld environment. Moreover, interactions among students and researchers enhanced students' work and promoted conceptual understanding.

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Childrens’ learning as participation in web-based communities of practice

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Abstract

Assuming learning as participation in communities of practice, and taking a situated perspective on learning, we studied children’s practice within WebLabs activities. The data collected was video recordings of groups of children in selected sessions and material they published in their WebReports. Data analysis enabled us to describe children’s practice in the project and find evidence of learning in the following categories:

(i) The emergence of a shared repertoire, including:

- vocabulary instantiated in their ways of approaching problems, questions, demands, challenges (using technology, programming and modelling), representing and sharing ideas (describing their work, ideas and thinking, commenting on other peoples’ work and ideas, building on others’ ideas to further their own, e.g. constructing the Randomness WebLabspaedia)

- an emerging valuing of crossing boundaries (both cultural as well as in specific knowledge domains, e.g. on Numbers);

(ii) The co-definition of mutual engagement. This is visible through:

- an emerging acceptance of the partiality of knowledge as a positive contribution to the knowledge of the community as a whole group and not as a sign of ‘not knowing’ things (e.g. the exchange of modes of proving that a certain ToonTalk robot produces a certain sequence in the Guess My Robot activity);

- an emerging sense of responsibility for the overall achievement, i.e. the joint enterprise where children feel that they have a voice (e.g. by contributing to the improvement of the software, children experience a strong sense of belonging to a project team);

- an emerging sense of ability and pleasure in going deeper into their ideas and products (a kind of localized depth) by way of a set of conditions, namely: interaction with powerful computational tools, interaction with teachers and researchers who help sustain collaboration (acting as peers in the exchanges within their specific tasks) and possibilities for innovative representations.

Summary

In the environment we designed, students investigate mathematical ideas through exploratory programming in ToonTalk. They share and refine their ideas in group discussions in class and on-line discussions using the WebReports system. These activities are guided and monitored by teachers and researchers. All the actors – students across different sites, teachers and researchers, form the WebLabs community of practice (Wenger, 1998).

In this paper, we will focus on an activity called Guess my Robot, which is aimed at advancing students' understanding of number sequences. The activity is a game in which students invent a rule for a number sequence and model it as a ToonTalk robot (procedure) that generates that sequence. They then collect the first few terms of the sequence and publish them in a web report. Their peers explore these in their own ToonTalk environment. They use a variety of tools to uncover the rule of the sequence: ToonTalk programming, Excel and (even!) paper and pencil. Once they succeed, they respond to the challenge by posting a comment on the report, which includes a robot they created for generating the same sequence. This activity was implemented in 4 countries, by 5 groups of 4-10 students.

We will use this activity as a case study to examine the WebLabs project under the community of practice lens. In this activity we induce children to operate in a mathematical domain through the proposal itself, the tools available, the guidance provided by the teachers and the dialogue with peers and researchers. We are trying to cultivate and sustain communities of practice – both physically and virtually – where it is natural to make conjectures, test hypotheses, offer counter-examples and so on.

Our data includes the analysis of session videos, as well the products of students' work. Our presentation will discuss the co-definition of mutual engagement, learning as participating in the development of a practice and the emergence of a shared repertoire. In this summary we illustrate with two examples.

One of the themes that emerges from students' reflections on using ToonTalk to explore sequences is their growing ability to cope with complexity. As one student claimed: "it is very interesting that using simple robots we could create complex number sequences" children claim. This should not be seen as a sign that all is straightforward: "actually we do not like the fact that most of our predictions did not come true". We see here an emerging acceptance of the partiality of knowledge (e.g. the notion that one's prediction in science may not really be true), interpreted as a positive contribution to the knowledge of the community as a whole group and not as sign of simply 'not knowing' things. Evidence of this is given by the fact that when children were exchanging modes of "proving" that two "different" ToonTalk robots can produce the same number sequence, they went through sequential stages of approaching the idea of proving in mathematics, and the sense that it is the audience that gives (or does not give) credibility and legitimates the results.

We strive to understand the ways in which students learn by analysing (or tracing) trajectories of: the way they talk and refine meanings when they express their thought; the way they produce and adopt tools and artefacts; the forms they use to create representations, records and recall events; how they appropriate and invent terms and redefine old ones. This is the children's shared repertoire within their WebLabs practice. We see their vocabulary being instantiated in their ways of approaching problems, questions, demands and challenges.

The ways some children changed their forms of representation of programs to produce sequences is evidence of the changing and dynamic improvement of their repertoire. This is applicable both to the tools involved in Weblabs and to the mathematical concepts and processes. For example, children use several terms deeply connected to ToonTalk. Depending on the knowledge domain they are working on, they also share some other terms particular to the domain or to the key actions involved in that domain. For example in activities based round ecology, which we named 'Ecomodelling', children talk about "changing behaviours" referring to the modelling processes involved. In activities based around the idea of randomness, expressions such as predictable and unpredictable, "enough information", "choices", "extractions" are used by children and are part of their regular vocabulary both in the sessions and in the webreporting. In Numbers activities, terms such as "generating intelligent and challenging number sequences", "prediction", "complex sequences" are part of their daily discussions and reports. Similarly we identified ways of representing ideas that become part of childrens' repertoire, such as including a box with "instructions" (e.g. "Read Me") in the set of boxes with robots and inputs in order to make easier for their imagined audience to use (peers in other schools and countries).

References

Wenger, E. (1998). Communities of practice – learning, meaning and identity. Cambridge: Cambridge University Press.