THE WEBLABS PROJECT: BUILDING NEW FORMALISMS FOR MATHEMATICAL AND SCIENTIFIC IDEAS

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This symposium will report the findings of the WebLabs project, a three-year research study funded by the European Union, investigating students' modelling of mathematical and scientific ideas. The fundamental idea of the project is twofold. First, to design and build "transparent modules" (TMs), carefully packaged sets of tools with which students can construct working models of their evolving knowledge in the specific domains chosen. TMs are *modules* in the sense that each has embedded within it a set of mathematical and scientific ideas that are operationalised in the form of working models. They are *transparent* in the sense that it is relatively straightforward to inspect the mechanism that makes the modules work, to manipulate and change them, and to rebuild them as necessary.

The second component of WebLabs is a "WebReport" system, which serves both as the collaboratively-constructed, public record of the evolving understandings of a knowledge domain among the community, and as the final product of the community's work. The idea is that WebReports afford a way to share working models built with the TMs that frame discussions, provide the language by which disagreements, conjectures and putative 'proofs' can be resolved and formulated, and ultimately lead to a co-constructed and consensually validated group report.

The presentations will focus on design issues in the development of the system, together with data derived from students aged between 13 and 15 years engaged in exploring Infinity, Sequences, and Force & Acceleration. The symposium will conclude with an overview of the theoretical and practical lessons from the project. Our approach is based on inviting students to use ToonTalk (www.toontalk.com) in order to program models of their evolving ideas. ToonTalk is a fully functional, concurrent programming language (Turing-complete) that has an interface modelled in the style of a video game: it is not text-based. Our approach forms part of a more general challenge to design and build systems that provide alternatives to the usual formalisms necessary for the study of mathematical and scientific ideas (Kaput et al, 2002; Noss, 2001).

References

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WEBBING CONNECTIONS BETWEEN STUDENTS' UNDERSTANDINGS OF POSITION AND VELOCITY GRAPHS

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In this paper we will share some findings concerning how students controlled virtual models of moving objects, analysed the interrelationships of position, velocity and acceleration and shared their models and analyses over the web. We have previously reported on a similar design approach in a different area – modelling 1-dimensional collisions (Simpson et al, 2005).

We focus on three main activities. In the *Green cars* activity, students watched a simulation of a moving car with corresponding speed and position readouts, predicted the position-time and velocity-time graphs, plotted the actual graphs in Excel from recorded data, and compared with their predictions, explaining similarities and differences. In *Guess my Graph*, students at one site controlled the motion of an object in ToonTalk to produce a kinematics graph in Excel, and then posted their graph on our WebReports site as a challenge for students at other sites to reproduce. In the *Matching Representations* task, students worked together in groups to match the corresponding narrative, position-time graph, velocity-time graph and ToonTalk descriptions of the same motion event.

Observation of students' dialogue, modelling, and reports suggests that they developed good understandings of the interrelationships between position, velocity and acceleration and the corresponding time graphs, concepts which are well-known to be difficult (Beichner, 1994). The key insight we derived was that the programming allowed students to shift representation by connecting to the 'real' situation they had modelled: the programs acted as a process-oriented link for developing the visual representations.

From the point of view of learning outcomes, students' overall correct answers in the pre- and post-test multi-choice questionnaire improved from 29% to 50%, although we do not claim statistical significance owing to our small sample size. More compelling is our qualitative data which suggests that students became highly engaged in much of the modelling and discussion. We will illustrate with video samples.

References

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A MULTIPLE-REPRESENTATION APPROACH TO EXPLORING SEQUENCES AND CONVERGENCE

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The notion of convergence of sequences is a well-documented stumbling block for students of all ages. For instance, many first year undergraduate students continue to believe that a sequence cannot reach its limit or that the limit is the last term in a sequence (Eade, 2003). Although most of the literature focuses on difficulties with the formal ($\varepsilon - \delta$) definition of limit, Tall & Schwarzenberger (1978) suggest that the root of these difficulties may lie in an uneasiness with the notion of infinity "*as if it were all a piece of mathematical double-talk, having no real-life meaning*". The research reported here aims to address this issue, by designing a set of activities, and the appropriate tools, which help students construct an intuitively-based approach to sequences and convergence without the difficulties inherent to the $\varepsilon - \delta$ formalism.

In this presentation, we describe a design experiment conducted as part of the WebLabs project, in which students aged 13 and 14 were able to investigate questions such as: Can a sequence get smaller and smaller and never go below 0? What happens when you sum the terms of such a sequence? How can you describe the differences between sequences, and how do these differences affect the convergence of the corresponding sum series?

A particular feature of the work was the interplay between standard and novel representations: in particular, how the ToonTalk tools (specifically the *Streams* pattern, borrowed from computer science) were coupled with Excel graphs and written texts to encourage a dual view (Sfard, 1991) of number sequences.

We will present some episodes that illustrate how students developed a rich vocabulary for describing sequences, and were able to articulate sophisticated mathematical arguments regarding their convergence and divergence.

References

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MAKING INFINITY CONCRETE
BY PROGRAMMING NEVER-ENDING PROCESSES

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Great minds ranging from Zeno to Galileo found the concept of infinity puzzling and difficult to understand. A deep understanding of infinity had to wait until Cantor's discoveries in the second half of the nineteenth century. Cantor's theorems that the set of all integers, the set of all positive integers, the set of all positive even numbers and even the set of all rational numbers all have the same cardinality (while the set of real numbers between 0 and 1 is of a higher cardinality) are not usually introduced to students until advanced high school or university mathematics courses.

The general assumption that infinity is too abstract for young children is hardly surprising. There are deep mathematical waters to be navigated, and it may seem impossible to make much headway without an appropriate formalism with which to express ideas. The existing formalisms are static and difficult for most students (see, for example, Monaghan, 2001; Tall, 2001; Tsamir, 2001).

This paper describes an attempt to help children approach these ideas, by providing them with an appropriate alternative formalism with which to think and talk about ideas like these. Our hypothesis is that via carefully-designed computational explorations within an appropriately constructed medium, infinity can be approached in a learnable way that does not sacrifice the rigour inherent in the concept. The curious child can learn some deep, interesting, and different mathematics without first having mastered more advanced mathematics.

We will describe how children explored concepts of cardinality of infinite sets by interpreting and constructing computer programs in ToonTalk. Children programmed infinite or non-terminating processes that produce infinite sequences including the natural numbers, the even numbers, the integers, and the rational numbers. They show constructively the one-to-one correspondence between the corresponding sets of numbers. Our field studies have supported the hypothesis that children can build useful intuitions of infinity by constructing and manipulating infinite processes and the computational objects that hold the eternally growing sequences produced by these processes.

References

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