

Reflections on John Monaghan's "Computer Algebra, Instrumentation, and the Anthropological Approach"

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In reacting to John Monaghan's paper, "Computer Algebra, Instrumentation, and the Anthropological Approach," I have interpreted my task as examining issues and raising questions primarily from the perspective of a researcher, but also from the perspective of a curriculum developer, a teacher, and a learner (not all of which are necessarily distinct from each other). When examining the paper from one or more of these perspectives I continually ask how the ergonomic and anthropological approaches might contribute to advancing research, curriculum development, pedagogy, and ultimately the learning of mathematics.

An important clarification that John provided in his introduction was that he was an "outsider" giving his impression of the French computer algebra system (CAS) work. It is important to note that I also am an "outsider" to the French work. Hence, I will be speaking from an "outsider's" viewpoint about the views of an "outsider" to a particular body of work. I hope that the "distance from the original work" inherent in my viewpoint contributes positively to the critique, rather than serving as a barrier to meaningful discussion of both the French work and the points John raises about that work.

Some general observations

Monaghan reports an important historical point about the French research team led by Michèle Artigue addressing "research questions concerning actual use of CAS (as opposed to desired use)" (p. 2). That was a noteworthy aspect of the French work, and it seems that the need for additional research that studies mathematics students' (and teachers') classroom use of CAS is still quite important. CAS researchers today need to build empirically a basis for theory and test theory that will (a) inform researchers as well as teachers and curriculum developers about students' uses of CAS and (b) inform researchers and teacher educators about teachers' uses of CAS.

Monaghan notes that the potential of CAS initially was seen as freeing students from manipulations and focusing on concepts and complex problem solving. He later makes an important point that describes a more extended impact of CAS use—it often "involves a totally different set of mathematical actions" from what is required when using standard paper-and-pencil algorithms. This is consistent with the first part of Hoyles' (2003) statement that tools both shape students' actions and are shaped by students' actions. One way in which tools shape students' actions is by making available to them entirely different actions from what would be available with standard paper-and-pencil algorithms. And, not inconsequentially, the availability and use of those "totally different" actions then requires students to interpret the results of those actions. Through such interpretation, and through tasks not even feasible without CAS, I would argue that tools also can shape students' actions by engaging them with *different mathematics* (Heid, 1989) from what they otherwise would work. CAS tools, as well as other technological tools, not only

shape students' actions by providing different *approaches*, they create opportunities for students to encounter different *mathematics* in the course of using those approaches. Different mathematics can be made possible in several ways, by (a) allowing students access to topics ordinarily reserved for students in subsequent mathematics courses, (e.g., beginning algebra students' exploration of exponential and logarithmic functions), (b) enabling students to perform symbolic manipulations that ordinarily would be too complicated or too tedious with paper and pencil (e.g., examining a sequence of products such as $(x+1)(x+2)$, $(x+1)(x+2)(x+3)$, $(x+1)(x+2)(x+3)(x+4)$, and $(x+1)(x+2)(x+3)(x+4)(x+5)$ as illustrated in Heid, 1989), or (c) providing access to a new mathematical topic (e.g., chaos in discrete dynamical systems).

John's paper draws on or addresses a number of substantial conceptual notions in explaining or clarifying others. Among these are: ergonomic and anthropological approaches; principle-led research; the technical-conceptual cut; task, technique, technology and theory; artifact; instrument; instrumented activity; instrumentalisation; instrumental genesis; gestures; techniques; schemes; cultural, physical, and semiotic tools; instrumented techniques; instrumental orchestration; internalization; and privileging and appropriation. It is not easy for one paper to incorporate and relate these in an understandable way; John's paper has done a good job of clarifying, using, and raising issues related to these constructs.

When stating that "maybe theoretical issues are relatively unimportant for practical mathematics educators," Monaghan raises an important point that clearly merits discussion, although he chooses not to address it in the paper. Even though currently available CAS may, as Monaghan notes, "present the user with everything s/he can do and thus constrain the user's potential to develop their own symbolism", that does not seem to merit an indictment of the usefulness of theory to practitioners. On the contrary, I believe that theory is important for practical mathematics educators, but such theory needs *interpretation* (both in research that is shared with practitioners and in curriculum materials used by practitioners) so that practical mathematics educators can see the relevance of theory to their work. If students use techniques and develop schemes in an environment that is constrained by what one might consider to be an oversupply of CAS capabilities, then theory that helps to explain and predict such use and development is useful for understanding how students and teachers operate *in that particular environment* as well as for hypothesizing how they might operate differently if CAS with different capabilities were available to them. Theory developed from studies set in a constrained CAS environment can help to suggest how CAS use might be structured differently with tools that have less built-in, tool-developer-determined structure.

In reference to task and technique Monaghan summarizes Lagrange's argument that bypassing techniques by using technology does not necessarily provide a route to conceptual understanding. John's summary states "novices progressively become skilled in techniques by doing, talking about, and seeing the limits of techniques. This eventually leads to a theoretical understanding of the topic." But what kinds of "doing and talking about" techniques help students "progressively become skilled" and what, other than fairly general notions of performing actions and reflecting on them, facilitates this

progression and “eventually leads to” understanding? (Some aspects of rich schemes that illustrate understanding are given in the “interpretive dimension” of Figure 1 of Lagrange [1999].). Research that can provide more detailed analyses of the development of understanding in the context of use of techniques continues to be needed. And, curriculum developers and teachers need to build on that research to make decisions about emphasis on technique. Monaghan also identifies this as meriting researchers’ attention and offers it as an item for further discussion.

An ambiguity/tension

One of the ambiguities/tensions that Monaghan identifies is that of potential multiple uses of the term technique—one with an agent-tool focus (Trouche, 2005) and one focused on a way of doing tasks (Lagrange, 2005). The question is, are these two uses of the term *technique* really different? Perhaps not, if one focuses on the agent-tool relationship through a set of observable behaviors (gestures) and the other focuses on the variety of gestures observable in students’ performance of a task.

Integration of the two approaches

Monaghan raises the question of the possibility of integration of the two approaches and notes that schemes in the sense of Piaget and Vergnaud might serve as barriers to such integration. I do not necessarily see that as being the case, but I would welcome more specific description of how schemes might serve as a stumbling block to the integration of the two approaches.

Several scenarios are possible concerning integration of the two approaches. First, one might conclude that the two approaches are, and should remain, distinct, with each offering somewhat different insights into the use of CAS tools. Researchers then might design their work based on one perspective or the other. At first glance, I tend to agree with Lagrange in that it may be best, at present, to think of the two approaches as each offering a specific contribution, but not to try to reconcile or merge them into a single approach.

Another possibility might be for researchers to continue to think of the two approaches as distinct but conduct research that draws upon both. Monaghan notes that this is possible and cites Artigue (2005) as providing rich descriptions of teaching and learning that draw on both approaches.

A third possibility (and the most interesting one) might be for researchers to attempt to integrate the two approaches as suggested by Monaghan. That leads me to ask what integration of approaches might mean. Would integration be a reconciling of the two approaches via some common language? Would integration of the two be accomplished perhaps by something that constitutes a broader, more holistic view of CAS tools into which both approaches fit? It would be helpful to speculate about what such a broader view might entail and how it might incorporate the two perspectives.

If a more holistic view that integrates both approaches were possible, then one also would need to know how research, curriculum development, and pedagogy might profit from

such integration. If an integration of the approaches were accomplished, what might be the nature of advances that might occur in terms of research methods, and ultimately, curriculum development, pedagogy, and learning? It would be helpful to hear John's view of what such advances might look like.

Missing Elements: Teachers and Affect

Monaghan notes that although teachers are not ignored in French CAS studies, the teacher is not generally the focus of that work. He makes an important point that neither the influence of the class of students on the teacher's actions nor the co-construction of socio-mathematical norms is accounted for in either of the approaches. My question centers on whether these should be, or even could be accounted for, given the foci of these approaches.

One possibility, taking a very narrow view, is that it simply is not necessary to attempt to address the influence of the class of students on the teacher's actions and the co-construction of socio-mathematical norms within either of the approaches. Or, perhaps the influences of students on the teacher's actions and the co-construction of socio-mathematical norms are too minor to be of consequence to understanding aspects of students' use of tools in either of the approaches. I suspect that neither of these is the case, but this might be worth discussing.

Another view is that although it would be important for these approaches to account for the influence of the class on the teacher and for the norms that develop, such accounting is not possible, given the foci of these approaches. It might be the case that although teachers are substantially influenced by the actions of the class of students and co-construction of classroom norms is important, both approaches simply are too narrow to include either of these. Is that the case? Perhaps not. The ergonomic approach acknowledges the importance of instruction, for example, "it is essential to attempt to understand what type of mathematical knowledge is at play in the perceptive processes of an efficient expert and how this is connected to more analytical processes. This is a prerequisite to efficient teacher intervention in the interaction process involving artefacts and students to be studied in Cognitive Ergonomy" (Guin & Trouche, 1999, p. 201). However, it would be helpful if the ergonomic approach provided a more detailed accounting for instruction. For example, how does the teacher facilitate the students' interactions about techniques (elaboration on the "doing and talking about techniques" to which I referred previously)?

Still another view is that it is both necessary and possible for these approaches to account for the influence of the class on the teacher and for the norms that develop. It would be interesting to hear, either from an "outsider" or from the French CAS researchers how one might construe either of these approaches as being able to address influences of students on the teacher or co-construction of classroom norms. One might also raise other issues that involve the teacher (e.g., classroom discourse) and ask whether either of these approaches addresses those issues as well. Guin and Trouche (1999) suggest the importance of the teacher in making connections explicit for students: "Many examples can be given where results may disturb students; consequently, the teacher should present

situations leading to reflection on the various results from different calculation modes. Connections between real and complex numbers, exact and approximate calculations are not perfectly clear and require mathematical knowledge” (Guin & Trouche, 1999, pp. 204-5). However, this seems to imply that the teacher needs to make the connections for the students, connections that only are possible with greater mathematical knowledge. Is that accounted for by the ergonomic and/or anthropological approaches?

Affect also is a major missing element identified by Monaghan. It seems clear to me that student beliefs potentially can have a substantial influence on their affective responses to CAS and their propensity to use CAS. An example of this arises in Lagrange (1999), in which the author notes that “technical work did not vanish when doing mathematics using Computer Algebra. Not all students welcomed the relief from the usual pen and paper skills: some students considered these skills as important for success in Mathematics” (p. 62). Beliefs that non-CAS-based skills are important for future mathematical endeavors certainly have a substantial impact on how and how often students use CAS. So, how might the French CAS researchers account for affect (e.g., valuing CAS use and/or valuing paper-and-pencil manipulation skills, enjoying mathematics when using CAS or when using paper-and-pencil) in either of their approaches? Monaghan argues against separating affect and cognition, much as is advocated by McLeod (1992). McLeod describes ways in which affective responses result from the cognitive evaluation of discrepancies between beliefs or anticipated actions and what actually results; cognitive evaluation of the discrepancy provides meaning to the affective response. Perhaps CAS research could use students’ beliefs and expectations and the deviations from those expectations that result from the outcomes generated by CAS use to incorporate affect into these approaches. I would welcome suggestions from the French researchers as to how this might be done, or whether it even would be wise to attempt to incorporate affect in either of the approaches.

Can the French CAS research improve mathematics teaching and learning?

Monaghan is somewhat skeptical about the universal applicability of the French CAS research: “I do not believe scholarship and research can be applied directly to further research or curriculum development outside the domain/country in which that research originated.” Based on my belief (not empirical evidence) that teachers and learners in different countries/settings are more alike than different with respect to the nature of their learning, I take issue with John’s pessimism about applicability of the ergonomic and anthropological approaches. He does note that research in one country or setting can inform research design, but I believe that it can do more than that. I believe that it can provide important basic constructs from which research and curriculum development can proceed elsewhere. (Of course, in some instances local considerations and constraints may impede applicability of research from other settings to research and curriculum development.) The French CAS research is based on a substantial body of work, is carefully done, and has no inherent impediments to applicability in other settings. The French research is of great enough importance that it *should* influence research and curriculum development elsewhere, as it, in turn, should continue to be reshaped by research done in other countries as well.

References

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