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"New technology? New ways of teaching – no time left for that!"

The title describes the two challenges which mathematics teachers nowadays have to deal with concerning their classroom-arrangements: include new teaching methods and integrate computers. Many teachers are afraid when realizing both trends makes curricular prescriptions even more difficult to achieve. Changes in the classroom-arrangement and integrating of technology are perceived as time supplement and an additional burden. In contrast to this other teachers perceive those trends not as an impediment, but as a special opportunity to achieve aims in terms of contents and processes. It was intended to investigate the question whether the combination of the both trends is an impediment or an opportunity in the frame of a research project at the University of Duisburg-Essen. In a first step teaching material was developed, which should serve for teachers as an example for a long-term-sequence about a compulsory topic with a combined focus on integrating CAS in an open learning-arrangement. Thus material for self-regulated learning to investigate polynomial functions in an open classroom-arrangement integrating CAS has been created. In a second step this material was evaluated. The central question of the research was to investigate to what extent this learning arrangement is suitable for simultaneously pursuing aims in both content and process.

According to the multi-faceted arrangement, a complementary research design that collects qualitative and quantitative data was chosen. The qualitative part is an interpretive study based on video tapes. The quantitative part is an experimental large-scale study. The material was used in 45 classes (approximately 1200 students) from different schools in order to check if general conclusions can be drawn. The large-scale study also includes a post-survey and a comparative post-test. To understand the aims of the project it is necessary to grasp the idea of the material. Therefore chapter 1 points out the main ideas of the material, chapter 2 explains the focus of the research project and in chapter 3 you will find early results that are organized according to teachers' learning.

1. The development of the teaching material

At first the material will be presented by a short overview to give an impression of what it looks like. Afterwards some theoretical aspects will be discussed to explain the decisions concerning the topic, the classroom organisation, and the use of CAS.

1.1 A learning workshop

The material is meant to be used as an introduction into the aspects of the investigation of polynomial functions with aspects of differentiation (like slope, zeroes, extrema, inflection point). Students of 11th grade (approximately 17 years old) receive a work folder on paper (Barzel / Fröhlich/ Stachniss-Carp 2003) with a set of worksheets (or set of "modules") that they deal with independently in groups of 4-6 students for approximately 6 weeks[1]. Supplementary material concerning individual stations is laid out in the classroom. The whole organisation of the teaching is like a work-place or a circle with different ways and possibilities of approaching the topic. This kind of classroom-arrangement is called a "Lernwerkstatt" (translated: learning workshop).

The only previous knowledge the students must have is the idea of derivation. It is possible to proceed through the learning workshop in different ways of orders. Different ways of learning are also usually possible within each module. There is no sequential arrangement of tasks in this case and the suggestions are given as mindmaps instead (for example module E is given as a mindmap - see fig. 1). At several points in the learning workshop, a comparison of the different types of representation of a function (graph, term, table, situation) is taken as the theme and the advantages and disadvantages are discussed. The example shown in figure 1 is from Module E (topic: extrema).

[1] Some further information appears in: http://www.lonklab.ac.uk/came/events/reims/3-ShortPres-Barzel.DOC

A variety of different types of tasks is involved in the material to evoke different kinds of student activities, for example:

- tasks which demand open ended approaches (Becker/Shimada 1997, Herget 2000),
- tasks which stimulate discussions between the students,
- tasks which initiate flexibility between the different representations in different modules to address different learning types (Herget/ Jahnke 2001) (see module E, fig. 1), and
- tasks which integrate students´ own experiences and experiments (Barzel 2002).

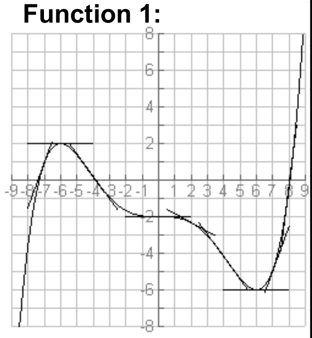
The following types of tasks should give an impression of this variety of the tasks across the set of modules:

Giving functions with concrete analysis assignments: This is performed for example in module E (see fig. 1). Three different functions are given, one as a graph, one as a table, and one as a formula. Without further previous knowledge concerning extreme points and their properties, students have to recognise the properties by analysing the three examples. In module L (higher derivatives) graphs of a function and its derivatives are analysed and connections between the degree of the polynomials and the maximal number of zeroes, extrema, points of inflection are recorded in structured form in a table.

Discussion of statements: If you concern yourself critically with a predetermined statement, you reflect on and interlink knowledge already acquired in order to arrive at an appropriate assessment. For example, assessment of the statement "$f'(x_0) = 0 \overset{?}{\Leftrightarrow}$ An extremum exists in x_0" results in development of an arithmetical method for determination of local extreme values (module "E" – see fig. 1).

You can see three functions in different representations. Determine the local extrema and try to define the concept "local extrema". What are the benefits and problems of the different representations?

Function 1:



Function 2:

$f(x) = x^2 + 2$
mit $-2 \le x \le 2$

Function 3:

x	y
0	13,5
1	5,94
2	1
3	-1,69
4	-2,5
5	-1,81
6	0
7	2,56
8	5,5
9	8,44
10	11
11	12,81
12	13,5
13	12,69

Find a calculation to determine local extrema. Use this calculation for the functions given by the following equations.
Check by plotting the graphs.

$f(x) = \frac{1}{3}x^3 - x$ and $g(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$

Why is the adjective „local" important?

„If the first derivative is 0, then there is a minimum!" – Discuss this statement and correct it if necessary.

Fig 1: One module as an example: Module E - Extrema

Text analysis: In module L (higher derivatives) information concerning higher derivatives is given. Beside that a research on the topic of inflection point is required by the task: “What is an inflection point? Inform yourself!” By means of structuring and separating important and unimportant aspects, dealing intelligently with mathematical texts is practised.

Experimental experience: “Derivative graph walking” (Module A) is a module that encourages trials with a sonic motion detector (CBR – “Computer Based Ranger” connected to a TI calculator). Movements are recorded indirectly as a time-distance or time-velocity diagram. A graph of a derivative is produced in this manner by one’s own walking. This type of graph is analysed and conversely, students try to walk to match predetermined graphs. As a result, cognitive discussions are linked with concrete experience, in order to facilitate comprehension of the new contents.

An overall reflection of the workshop is finally performed by preparing posters for a final presentation.

To give teachers an idea of how to use the material in their classroom teaching, an introductory booklet serves as a guideline with main ideas and recommendations for realising the workshop in their own teaching. The booklet also contains additional material for laying out in the classroom (e.g., two games). One recommendation concerns the documentation of the learning process in a kind of student learning journal, in which a student can write not only tasks and results, but also ideas, individual examples, meanderings, highlights, etc. (Hußmann 2003, Ruf/ Gallin 1998). In order to evaluate the learning process, the teacher has several possibilities: s/he can assess the group work by judging the individual student's participation and engagement during the group work and by assessing the student’s presentation of results (by poster or other visualisation). In addition, the teacher can check the learning journals of the individual student and, of course, the results of the student’s final written test.

1.2. Theoretical aspects concerning the topic

As the teaching material was supposed to convince the “average” teacher to think about a change of his/her teaching, the creation of the teaching material was firstly a matter of clarifying the question of the topics based on which the example was to be set. The following reasons led to the choice of the topic “investigating polynomial functions”:

- It is a mandatory topic and not an additional teaching topic.
- It is a topic on higher secondary level, where teachers see usually a more appropriate place for CAS than in lower secondary level.
- The topic “investigating a function” is quite often taught as a fixed procedure which has to be done in certain pre-determined steps with drawing the graph of the function at the end. This scheme provides much opportunity for criticism and is above all perceived as unsatisfactory by the teachers themselves. The core of the criticism in addition to the lack of satisfaction lies in the fact that the underlying mathematics is not understood by the students. Instead they often blindly follow a certain scheme and use formulas.
- “Functions” as a mathematical topic is a wonderful example of showing the benefits of involving CAS into the learning and teaching process. CAS offers the possibility of representing a function in different parallel ways (as an equation, a table, or a graph), thereby allowing an interactive alternation between these different mathematical representations. This “change of windows” (see the “Window shuttle principle”, Heugl/ Klinger/ Lechner 1996, The “Rule of Three” – analytically, graphically, numerically by Hughes-Hallett (1991)) can be

rendered useful for the learning process, since the different preferences of individual students can be addressed specifically. This interaction among representation types can be not only of benefit in devising an investigation of functions, but also useful whenever functions are dealt with in a school context (Tall 1996).
- The topic gives rise to many and diverse problem statements with which diverse activities in the learning process can be stimulated.

The central idea of the concept was to find a balance between instruction and construction, achieve an equilibrium between endeavouring to convey requested curricular contents and the desire for maximum opening up of the individual problem statements in order to stimulate many diverse and intensive debates on the part of the learners. It was intended to initiate a range of cognitive activities between receptive and creative activities.

What does this mean however in concrete terms within the context of the topic? Which activities are specifically meaningful and necessary in this case? An initial guideline for the realisation is first the description of "mathematical literacy" in the PISA framework (PISA 2000), the competence list in the educational standards of the KMK (2003) or the NCTM (2000), and likewise the hierarchical structuring of competences in the topic area "calculus" according to Tietze/ Wolpers/ Klika (1996). When creating new tasks for students these general categorisations have to be specified for the single topic area. Thus the catalogue of activities shown in Table 1 has been developed. The subdivision is in no way disjointed. It serves mainly as a guideline.

Table 1. Catalogue of cognitive activities

Type of activity	Mathematical work
Receiving activities	calculation, use of formulas, execution, listening, comprehension
Presentation activities	mathematical representation (in term, graph, table and words), switching between different forms of presentation, e.g. visualisation
Analysis activities	encoding of given texts and representations, interpreting, structuring
Reflection activities	comparison, rethinking a solution method from modified standpoints, interlinking
Creation activities	including new aspects, trying new viewpoints, devising and finding examples, systematising, generalising and investigating

Besides these general categories, metacognitive competences have also been taken into account.

1.3 Theoretical aspects concerning the classroom arrangement

In order to be able to develop all these activities, it must be possible to verbalise and communicate freely. Hefendehl-Hebeker (2004) points out the characteristics of a rational dialogue in dialogic learning. Such a dialogue should be unbiased, informal and non-persuasive to ensure that learning constitutes an act of self-controlled formation of networks of knowledge linked to one's own existing knowledge and abilities. For this purpose, however, a social constructivist framework must be created in which ideas and thoughts can be exchanged in an "uncensored" manner. Students must learn to get involved with others and to listen to their ideas in order to comprehend strange and perhaps unfamiliar trains of thought and to link others' thoughts with their own. Connections are established, differences pointed out, and difficulties elucidated in this manner. An organisational framework which provides order and orientation on the one hand and which gives learners the necessary latitude on the other hand can be helpful here. This is not possible with conventional teaching led from the front of the classroom with phases of work in pairs or groups that are limited in time, but requires a teaching method consistently focused on the student. An example of this method is a learning workshop. This method can be found in several German primary schools. One also can find learning workshops in secondary schools in Switzerland (Weber 1991). A learning workshop represents a place that – in the literal and figurative sense – allows a topic area to be dealt with in many different ways, while a joint product in common activity is developed. Historical models include for example the "laboratories" of Helen Parkhurst (1922). Pallasch (1997) points out four principles as characteristics of a learning workshop, which have also been the basis for creating this learning workshop:

- Principle of *participation*: All actors (students and teachers) should participate in forming the learning process.
- Principle of *structuring*: The work should be structured in a clear way to obtain an optimal transparency.
- Principle of *wholeness*: The idea is to present a topic as a "whole" by offering a big variety of different aspects and tasks.
- Principle of *balance*: The balance between a final product and the actual process describes the main idea of a learning workshop.

1.4 Theoretical aspects concerning CAS

CAS is beside spread sheets and dynamical geometry packages one of the general tools for mathematics education (Barzel/ Leuders/ Hußmann 2005, Fuglestad 2005, Drijvers et al. in press). These tools can be used for a wide set of tasks and be considered to be general purpose tools that are not useful for only a limited number of specific tasks – that is the character and as well the most important benefit of general tools. General tools are becoming more and more important for mathematics education and their use is compulsory or at least recommended in the curricula from different countries, for example in the curricula of Norway (KUF 1999) and Germany (LfS 2004). Important for this project is the authorization in Germany for use of general tools in higher secondary level. In some states the use of graphics and even CAS is compulsory and in others it is at least recommended. Only in one state it is not authorized to use neither CAS nor graphics on higher secondary level.

General tools allow students and teachers much more freedom to shape and modify how to use them than specific digital learning environments allow. In this sense the instrumental genesis when using a general tool is a very complex process, because the general tool as artifact can be transformed into a big variety of various instruments (Trouche 2002). This process is even more complex in the frame of open-ended approaches and in an open classroom arrangement. Drijvers & Trouche (in press) point out the special problem for the teacher to build coherent systems of the instruments in the classroom community. They describe the instrumental genesis as individual as well as social problem and have introduced the notion "instrumental orchestration" for the organization of the computerized environment. In this project the "learning workshop" as a teaching method can be seen as an organizational support for the teacher to come along with the challenge of orchestration.

One main point in this special orchestration is the request that CAS is always available for the students whether on a PC or on a calculator (like Voyage 200 or TI-89). It is the decision of the student whether to use the CAS or not and how to use it. Thus students can achieve the competence to decide themselves about the usage of the tool. This aim is pointed out in several publications (Leuders/ Barzel/ Hußmann 2005; Fuglestad 2005; Friedlander & Stein 2001).

According to the need to have tasks that initiate a variety of different cognitive activities among students, the tasks must also initiate a variety of different ways to use the CAS. There are special assignments that require specifically the use of CAS and other assignments for which solutions are possible without this technology.

For the tasks where CAS is required or helpful, CAS is used for:

- generating examples,
- calculating (solving equations, systems of equations, determining derivatives and single values),
- checking calculations and ideas, and
- visualising certain aspects. (See Dörr/ Zangor 2000.)

2. The research design to evaluate the material

The original aim of the project can be described by design science in the sense of creating material, evaluating it in the complexity of every day teaching, and further developing the material. The idea for the project came up in the frame of the teacher-training organisation T^3 Germany (Teachers teaching with Technology), through which teachers has been asked via an online-questionnaire about their requests and necessities concerning teacher training. One result of this questionnaire was the teachers' request to have more material for long-term-sequences for compulsory topics to get an idea of how teaching can change when CAS is involved. On the other side, teachers actually have to deal with the demands concerning standards and competencies and they are looking for ways to come along with these new demands. These two trends (CAS use and competence standards) often are seen by many teachers as independent requirements and two additional burdens. In contrast, other teachers perceive those trends not as an impediment, but as a special opportunity to achieve aims in terms of contents and processes. It was intended to investigate the question whether the combination of the both trends is an impediment or an opportunity. The communication of the results of the project in the frame of in-service-training is intended as post-activity after the research itself. The results should serve as an answer to the concrete requests of the teachers.

Evaluating "real-life" lessons in a multi-faceted and long-term arrangement requires both a clear focus of research and a complex research design.

2.1 The research question

The original idea of the whole project was to give teachers an example that pursuing curricular requests and using technology in an open classroom organisation are not contradictions but goals that can be obtained simultaneously. The research however does not focus on the teachers´ but on the students´ view and their cognitive activities to investigate the "value" of such an approach. The idea is to find out which activities really are promoted in such an open learning arrangement with an integrated use of technology. Thus the central questions of the study are: To what extent is this learning arrangement suitable for simultaneously pursuing aims both in terms of content and process? Which cognitive students` activities are promoted by it? The catalogue of cognitive activities in Table 1 (chapter 1.2) serves as a theoretical frame in this area.

2.2. The research design

A complementary research design was chosen which evaluated the teaching in a qualitative way and a quantitative way. In the pilot phase, the teaching material was initially tested by six teachers, discussed with those teachers, and afterwards further developed and published (Barzel/ Fröhlich/ Stachniss-Carp 2003). Furthermore, the questionnaires for the subsequent quantitative study also underwent piloting.

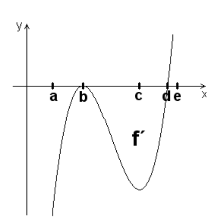
2.2.1 The qualitative part of the study

For the qualitative assessment**,** a class was monitored during the teaching with the learning workshop. Mainly the lessons were recorded on video, interviews with students and the teacher were conducted, students` learning diaries were analysed, and examination papers of the experimental class were studied in comparison to examination papers of parallel classes. Portions of the material were coded and analysed and assessed according to the guidelines of the Grounded Theory (Strauss/ Corbin 1996) – these include transcripts of teaching, students' exercise books, and the final examination paper of the experimental group (and the parallel classes as a reference group). In this process some lessons were chosen for analysis and interpretation which show a range of different activities inside small group discussion. As an instrument for interpretation, the epistemological triangle of Steinbring (2005, Bromme/ Steinbring 1990) has been used. In this model, mathematical conceptualising and meaning are described as mediations between signs or symbols and a suitable reference context, which is influenced by the mathematical knowledge and concepts as the third corner of the triangle. The whole process of conceptualisation is always a process which can only be described by series of triangles. For the specific purpose of this study the epistemological triangle as an instrument is supplemented by the perspective of cognitive activities. With this instrument the interactions between the students have been interpreted in a sense of meaning and in a sense of interacting in the communication process.

2.2.2 The quantitative part of the study

This part includes a post-survey (one for students and one for teachers) and a comparative post-test. For quantitative assessment**,** it was possible to recruit a total of 45 teachers (approximately 1200 students) in the school year 2003/04 in order to work on the topic with the aid of the learning workshop. This was the basis for an experimental large-scale study to look across schools to support broad generalisation. Both teachers and students were asked to complete a post-survey, and 578 students and 17 teachers gave feedback in this manner. These questionnaires enquired about individual attitudes to the work in the learning workshop, among other aspects, via statements that needed to be classified on a Likert scale.

f is a polynomial function over [a,e].
The graph of the derivative f ' is given in the figure.



a) Draw a conclusion about the grade of the derivative function f ' (Reasons!).
b) In which intervals is the graph of the function f increasing and in which decreasing (Reasons!)?
c) What characteristics shows the graph of f in b, c and d (Reasons!)?

d) Make a draft of a possible version of the graph of f.
e) Draw a conclusion about the possible number of zeroes of f in [a,e] (Reasons!).

Fig 2: Question 1 of the post-test

The participating classes took part in a final test. This test consisted of two questions (question 1 - see fig. 2), which were adopted from former central comparative examination papers in NRW (2002, 2001)[2] in order to select as the standard a requirement imposed from outside.

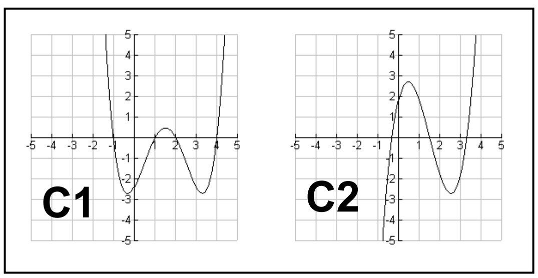
3. Some results of the study

3.1 Results of the qualitative study

One of the interpretive studies is about a sequence, in which a group of students worked on a game. They have 13 little cards on the table (C1 to C13)– and every card shows a graph. "What belongs together?" is the question to find three sets of graphs for f, f′ and f′′ among the 13 function graphs. In the analysed section, the discussion of two female students, P and U, is studied. Their different perspectives are apparent here. The first student mainly starts from a more global, dynamic way of looking at things, since she constantly moves the course and gradient of individual areas of the graph into a central position. The other shows a rather local-oriented approach, since she draws attention to points (extrema, zeroes and certain values) in a focused manner in order to discover sets of graphs.

P: *(points to graph C1)* ... the gradient is therefore in the negative range … the … is therefore, it falls and so ... y must be in the negative range for the 2nd derivative … *(P points to C2)*

…



U: There is a point of inflexion at this point here (*U points to C1, P points to C2, to a portion of the graph where the values of x are greater than 3*) and here is the extreme (*U points to C2*) \\ … and is extreme again here (*U and P point to C1*) and here (*U points to C2, the average zero point*) zero again (*P changes from C2 to C1*)

In the course of the discussion, both students increasingly get involved in the viewpoint of the other and interlink both local and global criteria in order to test, verify or negate the belonging together of graphs. This is increasingly successful, since their communication becomes more intensive and incorrect ideas are mutually corrected.

The experimental group (26 M + F students) and the three parallel classes from the same school (80 M + F students) took part in the same official central comparative examination paper in NRW (2003)[3]. All the students' solutions without the respective teacher's corrections were available to the group of researchers. In one problem, one equation of a function and graphs of four functions were predetermined and the task involved allocating the corresponding graph to the term. It was

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[2] It was respectively the 2nd problem of the comparative examination paper of 2001 and 2002, available at: www.brd.nrw.de/BezRegDdorf/hierarchie/lerntreffs/mathe/structure/sekundar2/vergleichsarbeiten.php

[3] This examination paper is to be found at: www.brd.nrw.de/BezRegDdorf/hierarchie/lerntreffs/mathe/structure/sekundar2/vergleichsarbeiten.php

apparent while correcting this problem that the students in the experimental group often began with the answer phrase and supplied the justification subsequently, unlike the reference group that began with justification and ended with the answer phrase. These and other characteristics were consequently quantified in order to detect any possible trends. The impression was initially confirmed in this case: 69% of the students in the experimental group placed the answer phrase at the beginning in comparison to 21% of the remaining group. In contrast, the phrase was at the end of the explanations in 19% of the experimental group and in 65% of the other students. Examination of the other characteristics provided possible reasons for this phenomenon. The control group usually began to perform calculations on the basis of the equation of the function (e.g., determining zero points and extreme points). These calculations were geared to elements that the students knew as components of the curve discussion. In the course of the work process the students found criteria for relating graph and term. The working direction was predominately from term to graph in this case. In contrast, in the experimental group, the working direction was more frequently from graph to term or moved back and forth between the two representations.

3.2 Results of the quantitative study

3.2.1 The Post-surveys

Both teachers and students were asked to complete a post-survey; 578 students and 17 teachers gave feedback in this manner.[4] About half of all the items have been identical or parallel in the students` and the teachers` survey and can be used for comparison. Parallel items refer to the same aspect in the students` learning process in both questionnaires – in the students` survey it is asked for the perception by the students themselves and in the teachers` survey for the perception by the teacher.

In this chapter some results of the analysis of the teachers` survey in general, and of the identical and parallel items in particular, will be presented.

General remarks concerning the teachers` responses

Only 17 of the 45 teachers gave their feedback via the online-survey. Three teachers wrote a letter and three teachers reacted by phone calls. These six teachers pointed out the same reason why they did not use the survey: For them the survey did not support the type of feedback they wanted to give. Those six teachers gave primarily positive feedback, but their reactions included mainly proposals for slight changes of the material and for correction of numerical faults in the booklet. But they all agreed to repeat this workshop in future courses and express their willingness to use similar workshops for other topics. Similar results could be drawn out of the answers of those teachers who filled out the survey, among whom 89% would like to use a learning workshop at least once a school year.

Comparison of the Likert scale items

In both surveys about half of the items were Likert scaled with given statements which had to evaluated on a scale running from *very strongly disagree* (---) to *very strongly agree* (+++).

Both teachers and students evaluated the whole arrangement in a positive way – teachers even more than the students. 83,3% of the teachers agreed to the statement *"The work with the learning workshop was positive!"* and 61,1% of the teachers prefer this kind of teaching compared with the classical way, where new topics are first worked out and explained in a

[4] The survey is available under: www.uni-essen.de/barzel/forschung.htm

plenary situation. This effect might occur from the circumstance that all the teachers who took part in the project have been teachers who already use CAS in their teaching.[5] Therefore these teachers can be seen as already engaged teachers and not as a representative group for average teachers.

The students` survey (n=578) has been analysed statistically by a factor analysis. This was not possible for the teachers` survey, because of the small number of returned questionnaires (n=16). The factor analysis of the students` items lead to reduce the dimension of the data to seven relevant factors (eigenvalue>1,3), which can be described by the following points:

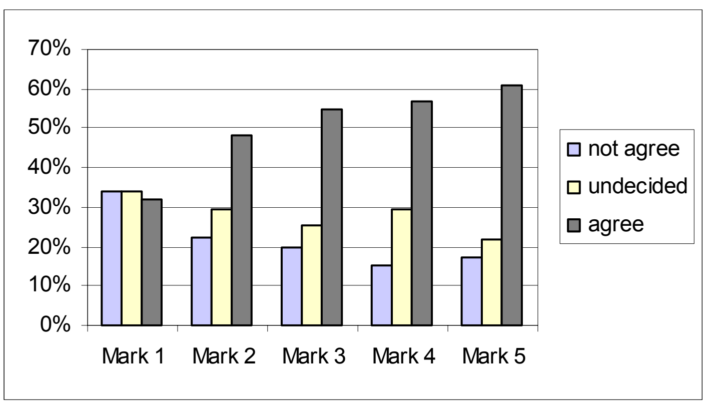
- Positive attitude to self-regulated learning
- Feeling confident with the content
- Positive view of documentation of the learning process in a journal
- Positive attitude to use of technology
- Higher work expenditure
- Positive attitude to the change of mathematical representations
- Further exercises

These factors and topics have served as a guideline to analyse the teachers` answers and to extract differences between students` and teachers` answers. Some selected aspects of the analysis will follow.

Positive attitude to self-regulated learning

Although the arrangement as a whole has been evaluated as positive, there are interesting differences between the teachers` and the students` perception when looking at special aspects of the self-regulated learning.

Teachers as well as students highly appreciate the possibility of students` informal speech in their own words during the long time of peer group work. One special item in the students` survey was the statement "It was helpful for me to speak in my own words without being corrected immediately." 51,3% of the students agreed with this statement and especially those students, which had weak marks in mathematics on their last report, showed a high proportion of agreement (see fig. 5)[6].



This result (in fig. 5) is conspicuous as long as self-regulated learning is quite often seen as difficult, or perhaps too difficult for weak students. A problem of such a long period of informal speech is that mistakes and uncertainty during the group work can not be corrected and solved immediately. Teachers show quite different opinions concerning this problem: 45% of the teachers announce that it is not good that students are not immediately corrected and 40% think it is good. This point seems to be a very important issue for further reflections and discussion with and among teachers

[5] Following a call in the online newsletter of the teacher-training-organization T[3] the teachers were selected.

[6] In Germany, 5 and 6 are the weakest marks and 1 is the best mark.

to transfer and work out further criteria for evaluating the communication between students doing mathematics.

Positive attitude to use of technology

Approximately 65% of all students and 73% of all teachers assessed the use of computers as meaningful and useful. A difference appeared here between female and male students: 73% of the boys and 59% of the girls – a result that confirms similar results in this field (Jungwirth 1992 & 1994, Niederdrenk-Felgner 1993). The use of computers also tended to be more positively assessed by those who used a pocket calculator or handheld with CAS (67,6% of the students) than by those who used CAS on a PC (61,6% of the students). A possible reason for the different attitudes to the use of computers may be the individually constant availability. Moreover, a handheld is outwardly less of a focus of attention and instead is "pulled out of the pocket" when needed and therefore can be more easily integrated in the learning process than a device that is more dominant because of its size and is to be shared with others under certain circumstances.

Positive attitude to the change of mathematical representations

Teachers and students assessed positively the following statement: *"It was helpful, that the different mathematical representations (equation, table, graph) have been involved so often."* 72,5% of the teachers and 54% of the students agreed to this statement, which shows the benefits of involving different approaches as Tall (1996) already pointed out.

Perception of students and teachers concerning the cognitive activities

Another format of items requested a selection of answers by making a cross in a list of given answers (multiple selections were possible in all those questions). This format was chosen for a feedback concerning the cognitive activities (fig. 4 shows an example) and for evaluating the several modules under different questions. For example students and teachers were asked in one item "In which module did you learn the most?", "Which module was most fun?" etc.

Which activities could you improve?

€ structuring of information
€ transcribing information (out of the schoolbook, of the neighbours documentation)
€ analysing; looking carefully
€ develop own solutions/ ideas for solutions
€ putting mathematics in own words
€ explaining to each other
€ repeating
€ looking up something (in schoolbook, internet, lexicon)
€ using the work and results of others
€ following thoughts of others
€ documenting and writing a journal
€ drawing graphs
€ speaking about mathematics
€ listening
€ organising the work

Fig. 4: Item concerning activities

Although it is finally not clear whether students and teachers have a clear and common notion of what is meant by the single activity, those items have been analysed because one could see tendencies of students' and teachers' perceptions. Adequate to the results mentioned above both groups pointed out in these items that students increased their communicative competencies. *Explaining to each other* and *speaking about mathematics* have been marked by a lot of students and teachers as well.

Teachers also realised an improvement concerning *organising the work (58%)* and *repeating (58%).* This was quite otherwise in the students` perception: Only 25% of them marked *organising* and only 8% *repeating.* Students might understand "repeating" only in terms of training skills and not so much also as an "inner repeating" such as filling out the table in module L, where you also can use – and repeat by this – previous knowledge about topics such as linear or quadratic functions.

Differences in perception can also be recognized by looking at the evaluations of the single modules. Two aspects should be highlighted here – the perception about the modules which are much fun and those where you can learn most. All modules with a game or an experiment have been seen as much fun by a higher proportion of teachers than students. For example 50% of the teachers marked the module with the CBR-experiment as a module with much fun in comparison to only 31% of the students. And under the question, *"Where did you learn the most?,"* 83,3% of the teachers marked module E (see fig. 1) and 11,1% module L. In comparison are the respective proportions of the students: 34,8% of the students marked module E and 39,6% module L.

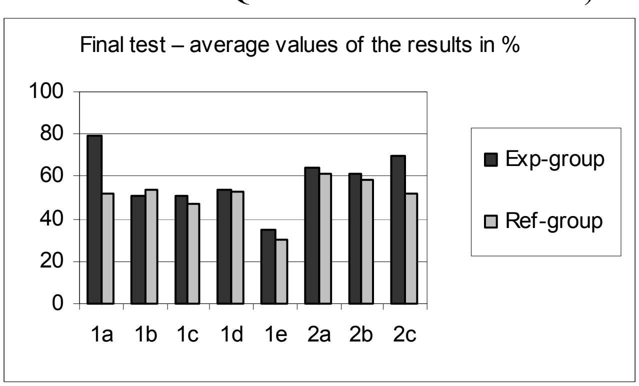
Module L requires mainly activities of analysing, structuring, and looking up something. Graphs of a function and their derivatives have to be compared and investigated and students have to inform themselves about "What is an inflection point?". Furthermore they have to use their results to fill in the following table:

Power of the function	Max. number of zeroes	Max. number of extrema	Max. number of inflection points

In contrast, the focus in module E was only on the concept of extrema by analysing three given functions. A possible reason for the different perceptions of teachers and students of these activities might be that students gain probably more certainty when connecting new concepts by finding real lines of connection – like building a web of knowledge as Müller/ Steinbrung/ Wittmann (1997) described an efficient learning process. Teachers are more used to a systematic way of teaching and inside this attitude a definition and a focus on one concept might be more clearly recognized as new information than structuring and finding connections between different things. This result is as well an interesting issue for reflection and discussion with teachers.

3.2.2 Comparative post-test

The participating classes took part in a final test with two questions (fig. 2 shows question 1), which were adopted from former central comparative examination papers in NRW. Thereby a requirement imposed from outside was used as a standard. For comparison, only the averages of the number of points achieved in the respective years in NRW were available. (12169 students (2nd problem) and 11364 students (1st problem)) The return of the final test amounted to 462 examination papers, corrected centrally at the university. The criteria centrally stipulated with the comparative examination papers were taken as a basis for the correction. Comparison of the mean results (in %) leads to figure 7 (Experimental group: 462 students; Reference group: Question 1 – 12169 students & Question 2– 11364 students)



Since only average values from the reference group existed, reference was also made only to the average values from the experimental group for comparison. After weighing all the influencing factors, it may be concluded from this result that the students in the experimental group satisfied the requirements of the central comparative examination papers at least as well as the students in the reference groups.

Fig. 7: Final test – average values of results in %

3.3 Final statement

The results selected should not obscure the fact that problems with an educational arrangement of this type might occur, particularly when the necessary openness in teaching is new and unfamiliar for both the learner and the teacher. Such a form of teaching requires abilities in a special way, specifically on the part of the teacher, in order to be able to use the variety and diversity productively in teaching. Nevertheless, the results obtained to date with regard to the central question of the study allow a positive assessment of the learning workshop presented. The observation that most diverse student activities are stimulated is a cause for hope that content and process aims are equally pursued in this manner. The results show as well not only a range of interesting fields for further research but as well interesting aspects for exchange with teachers in the frame of in-service-training. The learning workshop is already and will be subject of teacher training - not only as an introduction of the material or learning how to use a certain CAS but especially in the way to discuss selected results of the research project with teachers as a basis of reflection and development – of the material and of the own way of teaching. Of course this would be as well a good field for research….

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