Didactic time, epistemic gain and consistent tool: taking care of teachers' needs for classroom use of CAS.

A reaction to Barzel's “Open class-room? Computer algebra?... No time left for that….“

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Introduction

Barzel's paper focuses on "*students' use of CAS for open learning, their view and their cognitive activities*" in the MUKI project. MUKI’s goals and implementation also involved teachers working within the confines of curriculum requirements and time demands of 'real life' teaching. The aim of this reaction is to initiate from this project a discussion about teachers' needs for classroom use of CAS.

Barzel's argumentation is based on a positive view of 'general' symbolic tools, especially CAS. Although she mentions the "*very complex process of instrumental genesis*" and "*the special problem for the teacher to build coherent systems of the instruments in the classroom community*", she considers that "*these tools can be used for a wide set of tasks and be considered to be general purpose tools that are not useful for only a limited number of specific tasks – that is the character and as well the most important benefit of general tools*".

Barzel also reports a positive evaluation of the MUKI project, with respect to the students as well as the teachers:

> *The results obtained to date with regard to the central question of the study allow a positive assessment of the learning workshop presented. The observation that most diverse student activities are stimulated is a cause for hope that content and process aims are equally pursued in this manner.*
>
> *Both teachers and students evaluated the whole arrangement in a positive way – teachers even more than the students. 83,3% of the teachers agreed to the statement “The work with the learning workshop was positive!” and 61,1% of the teachers prefer this kind of teaching compared with the classical way, where new topics are first worked out and explained in a plenary situation.*

Teachers were brought to accept changes in their classroom activities and management, which appeared tolerable with regards to curriculum requirements and time demands and the majority of students and teachers seemingly appreciated these changes. I will not, in a first step, focus on the change consisting in moving towards more open activities. There is evidence that teachers can introduce technology especially CAS without changing much their way of teaching, while others are reluctant towards CAS because they see it as a threat for their open classroom management. For instance Kendal and Stacey (1999) report on two teachers involved in a TI-92 classroom project: one had an open classroom management using graphic calculators and was reluctant towards symbolic calculation, the other willingly integrated the symbolic capabilities of the calculator in his very 'instructive' way of teaching. I will rather look at what can be learned about teaching from the introduction of technology in the MUKI project reported in the paper, discussing the conditions that made the changes induced by this introduction acceptable for teachers. I will then consider changes in classroom management as a consequence of this introduction. In a last section, I will look at the extent to which MUKI's 'benefits' can be extended to other domains in calculus, discussing the design of CAS educational tools.

Teachers and CAS: a problematic relationship

In contrast with Barzel, many authors reporting about classroom integration of CAS stress that it was a problematic experience for teachers. Zbiek (2001) notes that experienced teachers have reached a certain ease in their teaching practices, which is disturbed by the introduction of computer algebra, allowing and demanding at the same time, new decisions, new actions, and a new understanding. Lumb, Monaghan & Mulligan (2000) report the successes and the problems met by two teachers, Steve and Stephen, when they tried to intensively use the computer algebra software, DERIVE. It is obvious that the introduction of a new technology requires the teacher to devote considerable time to lesson preparation, but according to Steve and Stephen, compared to other teaching software, DERIVE is at the upper end of the 'effort' scale. Moreover, it takes longer to get a 'feel' for how to use DERIVE, and the authors attribute this to the extensive possibilities afforded by this software.

Steve admits that many of his first ideas for exploiting DERIVE are actually not feasible. An analysis of the activities he deployed in class shows, among other changes, that he reduced the time he spent on what the authors call 'coaching' activity. 'Coaching' consists in engaging the students in thinking and reasoning about the mathematical situation under consideration, without telling them the key ideas. This reduction is paradoxical, since the introduction of computer algebra was supposed to allow more time to be spent directly on mathematical ideas. It is obviously related to the teachers' inability to 'feel' how to use the software for teaching and learning. Indeed, in order to elicit mathematical insights from students, the teacher needs to anticipate learning opportunities. Without technology, experienced teachers are able to improvise easily on their established strategies. In contrast, with technology and especially with computer algebra, it is not always as immediately obvious to the teacher how to exploit the situation mathematically and his previous ease in teaching is called into question. It is then easier for him to play the technical assistant than to explain about mathematics.

Steve and Stephen put a lot of effort into preparation of worksheets for students, deviating from their usual practice of using a textbook because they believed it was not possible to adapt much of the textbook material for use with the technology. Neither did they use many of the materials produced by the research group in which they participated. As stated by the authors of the article (p. 236): " *we think that teachers who plan to incorporate a significant use of computer algebra in their teaching are presented with a re-evaluation of the mathematics they were taught, and are familiar with*". Globally, Steve and Stephen tended to perceive trying to really integrate computer algebra as "*something that is neither rewarding nor desirable*" (Lumb, Monaghan & Mulligan, ibid., p.239).

MUKI's features likely to make changes acceptable by teachers

Trying to reconcile Barzel's paper to the above reports is relevant, because these teachers -- like the teachers in MUKI -- were involved in a project designed and supervised by researchers and were selected among volunteers with previous practice of technology. Care has nevertheless to be taken because the goals and the argumentation are dissimilar. Barzel's goal is to provide a 'proof by construction' that classroom use of symbolic tools can actually be compatible with teaching constraints without specially focusing on difficulties, whereas the above reports aim at elaborating from difficulties encountered by teachers in order to conceptualise the didactical challenge of integration. As Zbiek (ibid.) emphasizes, this approach to studying teaching may reveal, in a particularly clear way, the impact of computer algebra on teaching. Eventually, it may lead to the identification of particular strategies that make it easier for the teacher to teach with computer algebra.

I will assume that MUKI offers two features likely to make changes acceptable by teachers, and that analysing these features is an alternative approach to the observation of teachers trying to integrate technology. These features are:

1. consideration for the teachers' need to control the 'didactical time', and
2. consideration for the teacher's 'praxeological' needs.

Teachers' need for control over 'didactical time'

Barzel's title emphasizes teachers' concern for time. Underlying the expression "*No time left for that....*" is the teachers' worry that innovation, especially technology use, will bring them difficulties in time management. This worry is well-grounded: analysing an experiment of using Cabri at elementary school, Assude (2005 p.192) notes that teachers, "*repeatedly mentioned the discomfort of working in a hurry, without knowing what was coming next, the taking of risk, the radical change in how they planned and conducted their lessons.*"

Assude wonders why the teachers experienced discomfort and explains that "*to incorporate the work with Cabri into their geometry course outlines, the teachers adapted the outline of the previous year's course. But this adaptation did not necessarily allow the teachers ... to have a global vision...*" As the experience went on the second year, she noted that "*it proved to be an easier experience, since teachers could anticipate what pupils would be doing, and therefore they were able to manage their time more economically*". She concludes: " *In comparing the first and second years of our research, we noticed an increase in teachers' control over* didactic time"

What does 'didactic time' mean? This concept, a component of Chevallard's anthropological approach, is comprehensively explained by Assude (ibid p.184). I will just point out here that the time experienced by the teacher in the classroom is not just the 'ordinary' time counted down by the clock. For instance a teacher easily accepts to 'loose a little time' to greet students before starting the class in order to establish suitable conditions for learning activities, but (s)he is generally very unpleased by the 'waste of time' when students have been working on a task that appears to be irrelevant with regards to his(her) teaching aims. In the first case, 'ordinary' time is lost, but a gain of 'didactic time' is expected. In the second case 'ordinary' time goes on, but no 'didactic time' is gained.

What is 'controlling the didactic time' and why is it so important for teachers? Assude (ibid p. 193) explains: "*Control over didactic time implies tracing out a temporal division of knowledge that fits logically into the overall work of the class. Such control gives teachers a general view of how knowledge unfolds over time, even though it is possible to change things afterward, and makes it possible to anticipate pupils' difficulties during this progress.*"

Barzel's title's underlying idea is the loss of control over didactical time. While she points out (p. 1) that "*integrating of technology (may be) perceived as time supplement and an additional burden*" she does not specify how the MUKI project helped to overcome this difficulty. Searching for MUKI's features that might have helped teachers to keep control over didactic time, I found them in the care that has been taken to provide teachers with a comprehensive description of the teaching project.

Barzel says: "*To give teachers an idea of how to use the material in their classroom teaching, an introductory booklet serves as a guideline with main ideas and recommendations for realising the workshop in their own teaching.*" This 'LehrerHeft' is organized in 5 sections:

1 general information: contents, goals (mathematical, social, methodological and tool related competences), and students' introduction into the topic,

2 role and uses of computer tools,

3 descriptions of the individual components (contents, remarks including observation from previous experiment),

4 organization of instruction during the learning workshop, and

5 assessment of students' achievement.

In addition, the student document (SchülerHeft) explains by means of a diagram the several ways in which the individual components can be organised.

Certainly, this comprehensive description might have helped the teachers to identify the finalities of the project, to integrate them in their own goals, to understand how students' knowledge could progress over the time, to anticipate their difficulties and to plan the sessions while keeping the liberty to adapt their plans -- in a word, to keep control over the didactic time.

Teachers' need for new praxeologies

Beyond this control, teachers need effective means to make students work on a mathematical subject. Technology books offer problems that computer tools can help to solve nicely, opening deep insight into mathematics. For teachers these problems are often pointless. They have a curriculum to teach and, for most topics of this curriculum, well-tried strategies.

I offered to use the notion of 'praxeology' (the trilogy tasks, techniques, theories) to reflect on this situation. The notion of 'praxeology' is central in Chevallard's anthropological approach. Artigue (2002) and Lagrange (2005a) explained how it helps to think of the impact of technology on mathematical practices and on the study of mathematics, which is a specific practice. This idea is discussed by Monaghan (this symposium). Artigue and Lagrange focus especially on the role of techniques as links between tasks and theories, stressing that they are not to be seen just as routines (their pragmatic value), but also as means to understand the mathematical objects they involve (their epistemic value). Computer tools were designed to facilitate techniques and so necessarily have a strong impact on the technical level of mathematical activity, making new techniques possible and old techniques obsolete. Teachers' well-tried strategies can be seen as praxeologies efficient in the sense that the epistemic value of the techniques contributes to a mathematical understanding adequate for the curriculum. Technology most often trivializes these techniques, depriving teachers of their efficient praxeologies. This is a reason why so many teachers are reluctant towards using computer tools in classroom.

When they want to do it, they often have to rethink all their praxeologies, a hard task. Schneider (1999) offers an example where two teachers wanted to introduce students to TI-92 use in the study of logarithmic functions. They had to entirely rethink their teaching because the praxeologies they used to work with became obsolete. Without the TI-92, a central task was to solve exponential equations. Students progressively built techniques relevant for a variety of equations and learned about the properties of logarithms by reflecting on these techniques. The teachers became rapidly aware that the TI-92 solved exponential equations in one easy action and that all had to be rebuilt. The outcome was an entirely new approach to the domain; symbolic techniques were complemented by graphic and numeric exploration. It was certainly an interesting experience, but too hard for most teachers.

It seems to me that the MUKI project offers an example where the impact of computer tools upon techniques, rather than creating difficulties to the teacher, can respond to some of his(her) needs. As Barzel says, the general task -- curve discussion -- corresponds to "*an existing topic but perceived as unsatisfactory by the teaching staff themselves*". She points out that existing techniques are made "*only (of) a fixed sequence of pre-determine steps" (algorithms) and have a very weak epistemic value because* the underlying mathematics is not understood by the *students (practising this technique) and they often blindly follow a certain scheme and use formulas*". My interpretation is that a feature of the MUKI project is that it takes into account teachers' dissatisfaction with existing praxeologies to rebuild something not completely different -- a balance between the 'old' and the 'new', like Assude (ibid p. 184) would say -- but responding to teachers' aspiration for more epistemic value.

An example of a task and a discussion of associated techniques will help to explain this feature. It is taken from Barzel's paper (fig. 1 p. 2). The aim of the praxeology is to establish a balance between purely symbolic properties of extrema and the qualitative local aspect of this notion. The underlying mathematical fact is that for a polynomial[1] `p` and a real number `a`, `p'(a)=0` is a necessary but not sufficient condition in order that `p(a)` is a (possibly local) extremum.

In the description of the corresponding components, the MUKI teacher book draws on previous experimentation to point out that:

> *The traditional method of using the second derivative was used only by groups, which had "Wiederholer" (pupils who remain a second year in the same class) or if they adopted the procedure from the schoolbook. Anyway, this method is most difficult to understand*[2].

In my understanding this is a case of a 'traditional' technique existing in the German teaching/learning. This technique is based on a sufficient condition:

if `p'(a)=0` and `p"(a)` $\neq$`0` then `p(a)` is a (possibly local) extremum.

It can be extended by considering a necessary and sufficient condition:

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[1] Like in the MUKI project, I restrict the discussion of functions to polynomials in order not to have to make assumption on the differentiability of the functions. Obviously, tasks, techniques as well as the underlying properties extend to other classes of functions, provided that continuous derivatives of sufficient order exist.

[2] My translation.

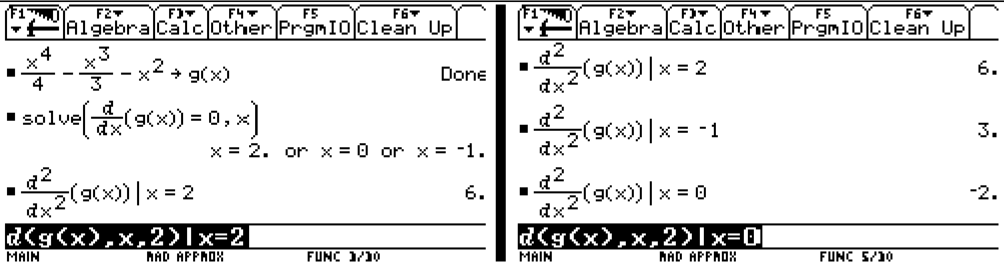
`p(a)` is a (possibly local) extremum if and only if there is an even number `n` such as `p(k)(a)=0` for `0<k<n` and `p(n)(a)≠0`.

This technique has a strong pragmatic value: provided that the successive derivatives and the solutions of the equation `p'(x)=0` are available, it is just applying an algorithm:

1. Compute the two successive derivatives of `p`.
2. Solve `p'(x)=0` in x.
3. For each solution, compute `p"(a)`. Conclude when this value is not zero.
4. If `p"(0)=0` then try other means (for instance the extended condition).

Without CAS, it implies of course possibly tedious symbolic calculation of derivatives and equation solving by applying symbolic computation rules. This algorithmic technique presents a weak epistemic value, because, as the teacher book points out, its justification, grounded upon the Taylor Young formula and properties of continuity is not accessible to 11th grade students. Furthermore, this technique focuses on a property of the function for isolated values, whereas 'local extremum' has to do with values 'around'[3].

What is the impact of technology on this technique? I would say that all depends on which technology. Computer symbolic computation alone 'trivializes' the algorithm by doing all calculations (figure 1). There is no epistemic gain.



$\frac{x^4}{4} - \frac{x^3}{3} - x^2 \rightarrow g(x)$ Done

$\text{solve}\left(\frac{d}{dx}(g(x)) = 0, x\right)$

$x = 2.$ or $x = 0$ or $x = -1.$

$\frac{d^2}{dx^2}(g(x)) \mid x = 2$ 6.

```
d(g(x),x,2)|x=2
```

$\frac{d^2}{dx^2}(g(x)) \mid x = 2$ 6.

$\frac{d^2}{dx^2}(g(x)) \mid x = -1$ 3.

$\frac{d^2}{dx^2}(g(x)) \mid x = 0$ -2.

```
d(g(x),x,2)|x=0
```

Figure 1: a computer symbolic technique to find extrema

In contrast, graphing or tabulating adequately complements the computer computation of the derivative's zeros by offering various means for local observation. The teacher book specifies these means:

- considering the graph, either with the 'trace' (kinaesthetic technique) or globally (pattern recognition technique),
- considering the derivative for x-values somewhat smaller and/or somewhat larger than the possible extremum (change of sign of derivative technique),

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[3] This technique is not taught in all countries. In France, the 'tableau de variations' (see Dana 2005) is established, after a global algebraic study of the sign of the derivative. This technique gives a wider view (but too wide: extremum appears as a 'global' rather than 'local' property).

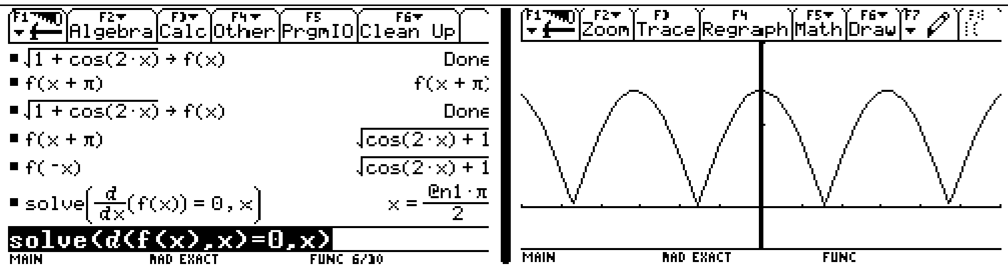
- using function values for x-values somewhat smaller and/or somewhat larger than the possible extremum (change of variation technique).

Technology then helps to build techniques combining a symbolic property -- **`p'(a)=0`**-- to a local observation of the function's behaviour. The epistemic gain is in a 'local view' on the notion of extremum as well as a better awareness that the symbolic property is not sufficient without 'local' discussion.

Another gain is that the technique is not algorithmic and offers interesting variations as shown above. The teacher book stresses that: "*When discussing during the poster presentation the different possibilities should be collected, explained if necessary, discussed and compared*". Indeed techniques are not epistemic in themselves. Their epistemic value does not actualise simply by practising. Students have to 'think about' issues like: what variation of the technique is best in a given case, what is the underlying knowledge... Classroom discussion, especially when taking place in special sessions where solutions are compared ('poster presentation' in MUKI), is a valuable means to trigger such reflective thinking. Mounier and Aldon (1996) experimented this when they transformed the problem of 'factorisations of x^n-1' in a 'long term problems' that is to say research conducted over three months. They give evidence that students had a deeper reflection about algebraic notions in the last session where they had to report on their findings. In this analysis, changes in classroom management are a necessity to take advantage of the new more varied techniques that technology can offer. They do not issue from general claims like 'teachers should change the way they teach' or 'constructive methods are better'.

Going beyond. The importance of design

This section discusses Barzel's assumption that standard CAS have a real potential to help teachers for classroom integration of technology. I will start from the following remark: if MUKI is a success from the point of view of the acceptance of technology by the teachers, it is because of the features discussed in the previous section, but also because the symbolic computation and the graphic/numeric representation of polynomials by computer tools do not contradict with what is expected of mathematical functions in secondary education. For other classes of functions, computation and representation by standard CAS is less easily consistent and thus can cause much trouble for the teacher. This is an example.



```
■ √(1 + cos(2·x)) → f(x)                 Done
■ f(x + π)                            f(x + π)
■ √(1 + cos(2·x)) → f(x)                 Done
■ f(x + π)                       √(cos(2·x) + 1)
■ f(-x)                          √(cos(2·x) + 1)
■ solve(d/dx(f(x)) = 0, x)         x = @n1·π/2
solve(d(f(x),x)=0,x)
MAIN          RAD EXACT          FUNC 6/30
```

Figure 2: Using the TI-92 to discuss a trigonometric function

The task was to study the function $f(x)=\sqrt{1+\cos(2\cdot x)}$. It was offered to 11th graders at the end of the year. The students used a TI-92 throughout the year within a research project (Lagrange 1999).

We expected that students would find easily periodicity and symmetry by observing the graph (figure 2, right) and by symbolic confirmation (figure 2, left), then detect that the derivative is not defined at the points where the curve reaches the x-axis by observing that the curve has opposed non-zero gradients at these points. (S)he would conclude that the function has an 'ordinary' (i.e. null derivative) maximum every $k\pi$ and a 'special' minimum every $\pi/2 + k\pi$.

Observation showed that parity was not a problem for students. In contrast, they had much difficulty to find a period, because of the phenomenon on the screen (left): the curve seems not to reach the x-axis for $\pm\ 3\pi/2$. After they detected that it was an artifact of the display, they had much difficulty to get a proper value for the period, because they could not make the link between the approximate value they got by tracing and the symbolic value of the period.

Interpreting accurately the behaviour of the function at the points where the curve reaches the x-axis was not possible. Students persisted to think of an 'ordinary minimum', wondering why repeated zooming in did not show a null gradient. They were reinforced in this idea by the false solution of the equation $f'(x)=0$ in the symbolic window, giving zeros of the derivative for every $k\pi/2$. Students had integrated this resolution as reliable means to get extrema and they had no reason to mistrust the result. Only one student detected 'something special' by trying to get the equation of the tangent line using the F5 menu of the graphic window and obtaining 'no solution'.

This observation shows first the limits of the help that graphing and tabulating can provide for calculus. It is certainly interesting that these students, like those in the MUKI project, think first to look at graphs when they have a function to discuss, but they have also to learn how to complement the graphic approach with a symbolic interpretation, not an easy goal to reach. The observation shows also the obstacle that CAS can create because of a non-consistent definition of functions: the TI-92 'does not know' that the derivative is not defined everywhere and then it looks just for zeros of the numerator.

This discussion of the use of the TI-92 to study the function $f(x)=\sqrt{1+\cos(2\cdot x)}$ is one example among many others[4] that standard CAS' design does not take into account the needs of secondary teaching. It is the direct consequence of a non-consistent definition of functions in standard CAS: in contrast with mathematical functions, CAS functions are just expressions without domain. This type of inconsistency is a major cause of teachers' difficulties in classroom. CAS are open environments where 'everything is available to the user' and students may obtain very varied strange or false results at any moment, creating a situation impossible to manage for teachers. Furthermore, it does not help students to 'stick' to the curriculum's notions, an enduring teachers' worry.

Barzel says: "*General tools are becoming more and more important for mathematics education*". The validity of this depends on what a 'general tool' is. Certainly open

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[4] Another curious example: the TI-92 gives a solution -1 for the equation of a real unknown x $\sqrt{x}\sqrt{x-3}=-2$.

environments have a greater potential than instructional software units. But there is evidence that their design has much progress to do in order to meet teachers' needs. In many countries including France, CAS use, even when it is allowed, is not encouraged and, anyway, it is used very little in classroom. Yerushalmy (1999, p.172) observes that "*the design (of CAS) serves the agenda of the tool designers – reaching a result in the smoothest possible way*" and this design contradicts with an educational agenda that should ensure tools' consistency with the curriculum.

With a group of teachers from the IREM of Rennes, after experimenting Derive and the TI-92 several years, we decided to tackle seriously the question of design and we initiated the development of a "symbolic calculator of functions", Casyopée. Lagrange (2005 b) explains

> *Casyopée's organisation is designed to help students to keep clear of erratic behaviour by concentrating on relevant objects in problem solving, to make sense of experimentation and to develop methods. As a difference with standard CAS, which operate mainly on symbols, each object has a clear status with regard to the curriculum: real number, function, parameter...*

A definition of functions consistent with the curriculum was a feature that teachers stressed most and then functions are defined on IR or on a union of intervals. This is how Casyopée can be used to study the function of the above example. Figure 3 is a copy of Casyopée's notepad. Unlike the TI-92, Casyopée does not give 'generic solutions' like **`x=@n1π/2`**, because theses formulations are too difficult to understand and manage for secondary students. The user defined the function **`f`** on a finite interval, where Casyopée is able to find a finite set of solutions for trigonometric equations. He then asked for the derivative. Casyopée detected 'forbidden values' and defined **`f'`** on a union of intervals. Casyopée gave then the right set of solutions for the equation **`f'(x)=0`**.

Unlike standard CAS, Casyopée's objects are entered or calculated by way of dialog boxes and buttons. They are displayed with their properties in specific windows (Lagrange 2005b). The notepad keeps a record of the entries and of the calculations.

This figure on the right is a copy of the notepad after

1. entering the function **`f`**,
2. calculating the derivative **`f'`**
3. calculating the solution of **`f'(x)=0`**.

```
Fonction définie sur  [−2π;2π]
f x → √(1 + cos(2 x))
______________________________
Fonction définie sur
[−2π;−3π/2[ ∪ ]−3π/2;−π/2[ ∪
]−π/2;π/2[ ∪ ]π/2;3π/2[ ∪ ]3π/2;2π]
f′          sin(2 x)
   x → − ───────────────
          √(cos(2 x) + 1)
______________________________
résolution f′(x)=0 en x
S = {2π, π, 0, −π, −2π}
```

Figure 3: a record of function definitions and calculations in Casyopée

Conclusion

Nearly ten years ago, Bottino & Furinghetti (1996 p.132) pointed out that "*The introduction of informatics in mathematics teaching works only when it is perceived as an answer to questions (even though unconscious) already present in teachers' minds.*" It has been a long time until innovators and researchers began to consider seriously teachers' needs for classroom use of technology, especially CAS. A contribution of Barzel's paper is that the MUKI project clearly took some care of these needs, complementing approaches based on the observation of teachers' classroom behaviour when using technology, outlined in section 2.

I elaborated from this contribution, trying to conceptualise from what is known of teachers' needs. Teachers absolutely need to have control over didactic time. It is relatively easy for teachers to do small activities using technology (one or two sessions), because control cannot be lost on so small a period. A survey in the French 'Instituts de Formation des Maîtres' (Blanchard & Lagrange in preparation) showed that during the year of training, which is also the first year of teaching, a majority of pre-service teachers do these activities, but do not go farther. A real integration would imply that they could have a global vision of how knowledge unfolds over time in order to anticipate what students will be doing and learning. Because it is really difficult in their first year of teaching, they tend to conform to 'traditional strategies', which do not include technology. Certainly projects like MUKI, not too ambitious but providing the teacher with adequate documentation, might help pre- and in-service teachers to feel more confident.

Teachers also need effective praxeologies. Technologies make new techniques available. Most often, these techniques endanger the delicate balance of teaching because well-tried praxeologies become obsolete. If, however, these techniques can be a basis for better praxeologies replacing existing non-satisfactory praxeologies, it is a strong incitation to change towards the use of computer tools. The case of the relationship between extrema and values of the derivatives in the MUKI project is a good example of an epistemic gain thanks to new graphic and numeric techniques. This aspect of the MUKI project also helps to focus on another condition for teacher acceptance of technology, developed by Assude and Gelis (2002): the need for a "right distance" between the old and the new. All tasks and techniques cannot be new, otherwise there would be too much to reconstruct for a teacher. In MUKI, tasks were familiar while techniques evolved towards a balance between symbolism and graphic/numeric exploration, thus teachers could integrate the changes in their teaching schemes.

Barzel (2003 p.1) begins an earlier paper about the MUKI project, by saying that "*The use of technology leads only to a deeper understanding if it is combined with a change of classical way of instruction to a more constructivist approach*". It is a strong statement, which puts much pressure on teachers. In this reaction, I pointed out that changing the functioning of the class by introducing workshops and students' presentations was means to take advantage of new, potentially more epistemic techniques. It is nevertheless a difficult reconstruction for many teachers. In my opinion, they are more able to do it when they can see actual advantages rather than under the pressure of general statements. Again as researchers, we have to think of a right distance between this 'new' functioning and the 'old' reliable teaching methods.

Because the MUKI project is about polynomials, standard CAS' use is not so problematic as it appeared in other observations. The acceptability of standard CAS by teachers cannot then be really generalized to other domains where there is evidence that CAS design has much progress to do in order to meet teachers' needs. Clearly, math educators cannot simply take tools designed with no awareness of secondary teaching's needs. They have to tackle the question of design, working with teachers on the development of symbolic environments to offer students Computer Algebra power but also consistency with the curriculum.

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