

Factors influencing the integration of Computer Algebra Systems into university-level mathematics education

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Introduction

Technology has become an important ingredient of mathematics and its teaching and learning. Although it has been predicted that technology integration into all levels of mathematics education would be rapid (Kaput, 1992), accumulated evidence indicates that incorporation of technology into education has been slow (Ruthven & Hennessy, 2002). Studies suggest that, beyond the accessibility of technology and policy pressures, teachers' beliefs and attitudes are vital factors influencing technology integration (Hennessy, Ruthven, & Brindley, 2005). Furthermore, international comparative studies demonstrated that teachers' didactical beliefs and conceptions of the subject, as well as the characteristics of their classrooms and their relation to technology, are heavily affected by teaching traditions and geographic locations (Andrews & Hatch, 2000; Beaton & Robitaille, 1999). However, studies, most focusing primarily on the school level, have rarely crossed national boundaries and have paid little attention to technology integration in university-level mathematics education. Due to the paucity of such research, little is known about the extent and manner in which technology is used in university mathematics classrooms. Thus, my study attempts to investigate the current use of technology together with the factors that influence its integration into university mathematics curricula. For my investigation I chose a specific computational tool, Computer Algebra Systems (CAS), which holds a prominent role position among software applications employed in university-level mathematics teaching and learning.

I developed a two-phase international comparative study to investigate the current use of CAS in universities, and the factors influencing CAS integration into university-level mathematics education. Specifically, my study aims to examine:

- the extent and manner of CAS use in university mathematics departments;
- the pedagogical and mathematical conceptions of university mathematics lecturers regarding CAS, including the factors influencing their professional use of CAS; and
- the extent to which nationally situated teaching traditions, frequently based on unarticulated assumptions, influence lecturers' conceptions of and motivation for using CAS.

In the past year, I conducted an exploratory qualitative study (phase I) to uncover issues related to CAS integration. This phase will be followed by a large-scale quantitative investigation (phase II) to further elicit issues with regard to CAS integration at universities. In this paper, I shall describe my methodological approach and outline the initial results of the first phase of the study.

Methodology and methods

Technology integration in schools is, as opposed to universities, has been extensively researched in many countries. School-level studies investigating the teachers' conceptions of ICT use have employed essentially two kinds of qualitative approaches. On one hand, American, Australian, British, French, and Italian researchers utilized a case-study approach to examine teachers' conceptions of as well as their practices with ICT (Bottino & Furinghetti, 1996; Laborde, 2001; Leatham, 2002; Monaghan, 2004; Stacey, Kendal, & Pierce, 2002). On the other hand, Ruthven, Hennessy, and Brindley in the UK (Hennessy et al., 2005; Ruthven & Hennessy, 2002) opted for a naturalistic grounded approach to elicit teachers' thinking about ICT. For the aims of the qualitative phase of my study the former, case-study approach, would provide a detailed set of themes obtained from a small number of teachers; however, the number of participants is insufficient to support a broader view of teacher thinking. In contrast, the latter, grounded approach, investigates a larger number of participants and attempts to uncover important issues that emerge from the data. Hence, the grounded approach delivers views of a sizable number of participants from a variety of institutions and supplies my study with a wider-range of issues. Therefore, I decided to adopt a

grounded approach employed by Ruthven et al. (2002, 2005) with modifications in my research instruments.

Following the guidelines of Grounded Theory methodology (Glaser & Strauss, 1967), I interviewed 22 mathematicians at a range of universities in Hungary, the United Kingdom, and the United States. In addition, I observed classes and collected course material during my university visits. The analysis of this data, involving coding strategies by Strauss and Corbin (1998), exposed a number of issues in relation to CAS integration at universities.

Results

The data analysis has revealed how lecturers' conceptions interact with personal characteristics and external factors. Figure 1 illustrates the interdependence of the clusters and the size of boxes indicates the number of issues which emerged in relation to each cluster.

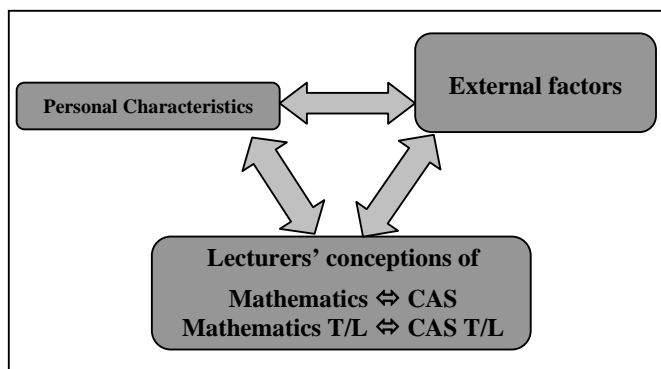


Figure 1

Each cluster is further divided into sub-categories as appropriate (a full list of the categories can be found in the appendix). Below, I offer a sample of the emergent issues, but the length of this paper does not allow me to provide a comprehensive description of all issues.

Personal characteristics

This cluster includes factual information about lecturers and about their work as mathematicians. These facts derive both from lecturers' self-reports and from the review of departmental documents and websites. Variables include: age; gender; nationality; type of university in which lecturers work; lecturers' research area; years of teaching experience; CAS use in research; CAS use in teaching; and computer proficiency. In addition to collecting facts about participants, I examined what conceptions lecturers hold about how the listed variables interact with CAS use. As a result, I discovered two conceptions held by lecturers that my data did not support.

Firstly, interviewees articulated a belief that researchers in pure mathematics fields are less likely to employ CAS in their teaching than those working in applied areas:

There are a few people within the faculty they think it is not wonderful and they would rather get rid of it. Those people are mostly on the pure side. [Dr Hamilton, UK]

In contrast to this belief, my data supply no evidence for differing use of CAS between pure and applied mathematicians.

Secondly, a number of lecturers perceived that young mathematicians are more likely to use CAS for teaching than their older colleagues, because young people are thought to be better acquainted with computers and they are less likely to change their teaching practices:

There is also some or a lot of resistance among the faculty ... [talking about older colleagues] ... because they don't see the point and they themselves are not comfortable with technology and they don't want to take the time. [Dr Wright, US]

Again, my data do not support this perception. Certainly, these observations are subject to further investigation.

External factors

In this cluster, I gathered issues that are beyond the immediate control of lecturers and those that may affect their working environment. This cluster is divided into two sub-sections: the first examines institutional and cultural environments of lecturers at different levels starting from the course-level to the national-level (figure 2); the second consists of issues that emerged in relation to technology.

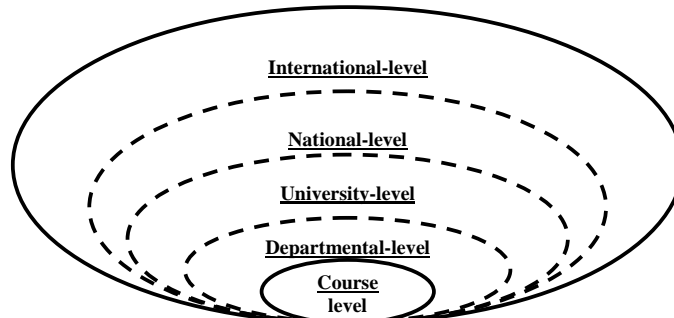


Figure 2

I have found a number of interesting issues with regard to lecturers’ environmental circumstances and about the advantages and drawbacks of technology use in education. For instance, lecturers pronounced that they do not experience pressure from their departments and universities to use technology in mathematics teaching; and they feel that various features of technology are not yet mature enough for immediate classroom use.

Internal factors – lecturers’ conceptions of mathematics, CAS, mathematics teaching/learning (T/L), and CAS teaching/learning

The preliminary data analysis suggested subdividing this cluster into four sub-sections based on different sets of conceptions. Initially, I attempted to investigate the four conceptions independently; however, during the data analysis, it became apparent that these issues are greatly interconnected and it would be difficult and superfluous to artificially separate these conceptions. For example, it is unrealistic to separate lecturers’ conceptions of mathematics from their views about CAS-assisted teaching.

Figure 3 illustrates how I interpret the relationship among the four conceptions.

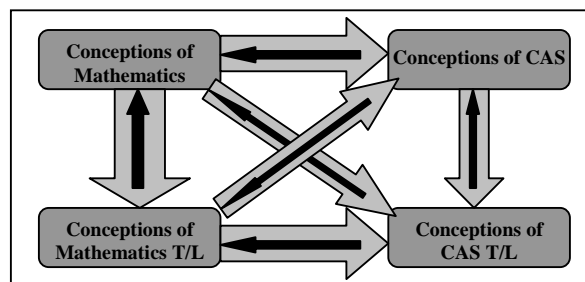


Figure 3

In the picture, the arrows indicate the tendency of conceptions to effect each other. It can be observed that all the four conceptions are connected, but the thickness of the arrows represents the strength of their effect. In accord with the literature, I observed that teachers’ conceptions of mathematics influence their other conceptions, and most significantly their conceptions of mathematics T/L, but that conceptions of mathematics are strongly related to teachers’ conceptions of CAS and CAS T/L. In fact, in the next section I shall explain in detail that lecturers’ conceptions

of mathematics have a more sizeable effect on their CAS T/L than was observed at the school level. (Full list of conceptions can be found in the appendix.)

Discussion

The emergent issues demonstrated much correspondence with results of school-level and international studies investigating technology integration. However, there are some differences that my study highlighted both between the school and university levels and among the participating countries. Prior to beginning the discussion, I must remind the reader that my sample is too small to permit confident generalization. Nevertheless, I can offer an insight into CAS integration issues at universities and reveal similarities and differences between my chosen sites in the participating countries. Below, I provide three observations that emerged from the data.

Conceptions

School-level studies suggest that teachers' conceptions of mathematics are consistent with teachers' conceptions of mathematics teaching (Thompson, 1992). Based on my data lecturers frequently talked about CAS and CAS teaching in relation to their mathematical views and principles. However, they less often referred to CAS-assisted teaching from a pedagogical standpoint. It is interesting to observe that whereas school-level studies revealed issues related to class management or pedagogy (Hennessy et al., 2005) I received little or no reference to such issues from lecturers. This may derive from the fact that lecturers are actively engaged in mathematical research, which requires rigorous mathematical reasoning and sound knowledge of their area of mathematics (Thomas & Holton, 2003). In addition, they may often establish their teaching practices on rigorous mathematical reasoning (Mura, 1993). Furthermore, teaching is not the primary engagement of lecturers' work. According to these observations it can be suggested that lecturers' conceptions of CAS and CAS-assisted teaching are rooted in their conceptions of deep and rigorous mathematical thinking and their conceptions of mathematics teaching play a lesser role in this respect.

So the idea here is that you learn the maths in this course, now you sit down and get a computer to actually do something ... So you are exploring simultaneously the computation and the theory and the output you get from the computer... It helps in two ways, one it helps reinforce the theories that you have done and helps you get used to the ideas and you feel more comfortable with it, and two it helps you see how the theory actually works out in practice and helps you visualize the ideas that were involved. [Dr Hamilton, UK]

Teachers' conceptions of ICT and ICT-assisted teaching are also rooted in their conceptions of mathematics, but their teaching conceptions play a more significant role in this respect than those of lecturers.

External factors

School-level studies revealed significant policy-rooted pressure on teachers to incorporate ICT into their teaching (Becker, 2000; Hennessy et al., 2005). On the contrary, my data suggest that university teachers enjoy far-reaching autonomy in their work. I did not encounter any instance where lecturers felt pressured, either by their department or their university. Furthermore, departments and policymakers respect lecturers' choices and rarely impose pressure on them.

... it is very difficult to actually to insist that somebody must do something in a certain way and even though I was head of maths I couldn't say to a colleague like when you teach this topic you must use the following package. You know because it is just the nature of the university and the academic freedom to do what how you want to do it. [Dr Cass, UK]

In accordance with this finding one might conjecture that lecturers' conceptions of CAS and CAS-assisted teaching are even more important influencing factors for CAS integration at the university-level than at the school-level, because external forces have little or no influence on lecturers' CAS related decisions.

Teaching traditions

The data revealed curricular differences between the participating countries particularly relating to the presence or absence of mathematical proof in the curricula designed for science/engineering students. However, according to the curricular changes in the participating countries, this difference has been diminishing in the past decades. In addition, academics participate in international conferences, collaborate across borders, take positions in other countries, and learn practices from each other, demonstrating that mathematics and its teaching has become an international enterprise at the university level (Atweh, Clarkson, & Nebres, 2003). This mobility has an effect on mathematicians' teaching practices beyond the teaching traditions they acquired during their studies and early career. Evidence suggests that teaching traditions still have an important influence on lecturers' conceptions, but this influence is less significant than that uncovered in school-level studies. Therefore, the sort of clear-cut differences identified by, for example, Andrews and Hatch (2000) at the school level should not be expected in any study conducted at the university level.

Figure 7 illustrates the argument that I outlined in the preceding paragraphs.

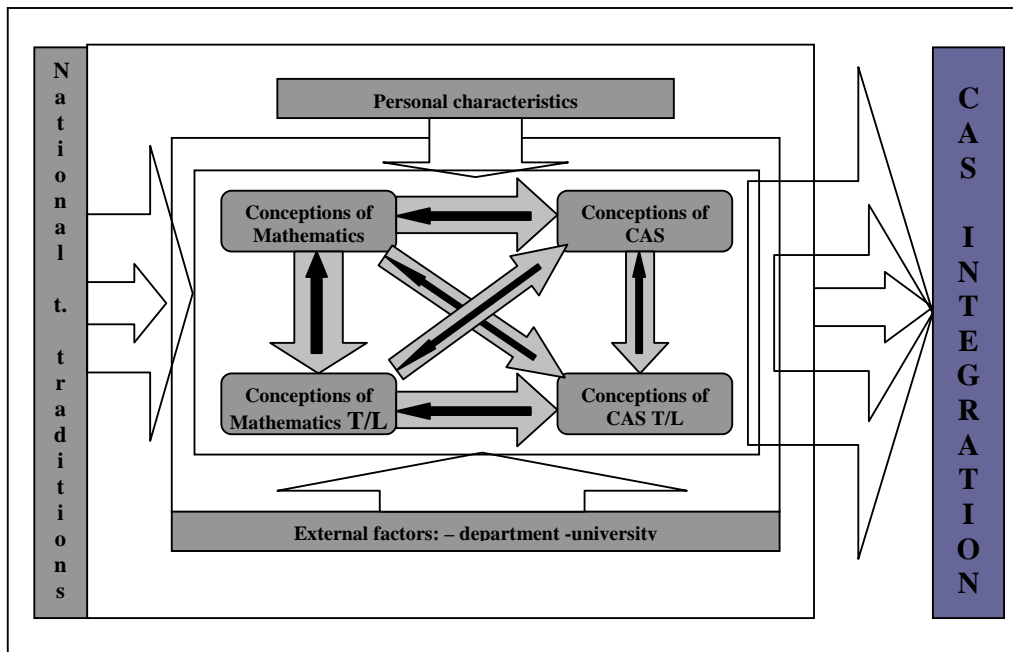


Figure 4

The arrows pointing to CAS integration represent the weight of influencing factors. It can be observed that lecturers' system of conceptions is the most influential factor with regard to CAS integration. It is followed by lecturers' immediate environment such as the department and university, then nationally-based teaching traditions. However, the latter two factors are filtered through lecturers' conception-system. In addition to these three factors the data revealed personal characteristics factors such as lecturers' research area and age that may also have an effect on CAS integration. Furthermore, I conjectured that lecturers' conceptions of mathematics affect most significantly the three other conceptions.

Summary

I made assumptions that issues found at the school level with regard to technology integration are only partially consistent with university-level findings although the evidence indicated some similarities. Firstly, I assumed that school teachers' conceptions of mathematics were likely to be less influential in respect of their computer use than at the university level. Secondly, I argued that that external pressure may have more dominant effects on school teachers' conceptions and thus on technology integration than those at the university level. Thirdly, due to university teachers' international mobility and acquaintances with international research and teaching, a cross-national comparative study would less likely provide such clear-cut differences in teachers' conceptions than

those established by school-level studies. In the second phase of my study, I shall further investigate the issues that emerged in the first phase and gather additional evidence to back my assumptions.

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Appendix – List of Emerged Issues

1) Personal characteristics

Variables

- Age
- Gender
- Nationality
- Type of university where lecturers work
- Mathematics research area (classified as pure/applied mathematics research)
- Years of teaching experience
- CAS use in research
- CAS use in teaching
- Computer proficiency

Themes

- **CAS use in research**
- **CAS use in teaching**
- **The relation of research area to CAS use in teaching**
- **The relation of lecturers' age to CAS use in teaching**

2) External factors

Institutional and cultural themes

- **Course level**
 - The nature/subject matter of a course determines the suitability of CAS use in teaching
- **Departmental level**
 - Differences of courses designed for a variety of student cohorts
 - Departments make decisions on their course offering
 - The department provides access to hardware and software
 - Departments' support of teachers in technology use
- **University level**
 - Universities provide ICT facilities
 - Types of universities - resources
- **National level**
 - The mathematical preparedness of students enrolling to universities is declining
- **International level**

Technology related themes

- **Critiques of hardware**
- **Critiques of software**

- **Encouraging advancement of technology**

3) Internal factors – lecturers conceptions of mathematics, CAS, mathematics teaching/learning, and CAS teaching/learning

Conceptions of Mathematics – Conceptions of CAS

- **CAS has become a part of mathematical knowledge and a new mathematical tool**
- **CAS use is affected by distinction of the mathematics done by mathematicians and engineers/scientists**

Conceptions of mathematics teaching/learning – Conceptions of CAS teaching/learning

- **Assistance in teaching**
 - Activities that can be assisted
 - Features that can be integrated into teaching
 - Requires time commitment
 - Role in sequencing teaching
 - Role in teaching
- **Benefit for students' future studies and work**
- **Assistance in learning**
 - Relation to the pace of learning
 - Skills students must learn
 - Relation to students' abilities
 - Environment distracts students from learning
- **Relation to assessment and examinations**
- **Role as mathematical tool**
- **Benefit for students' future studies and work**