THE USE OF CAS IN SCHOOL MATHEMATICS: POSSIBILITIES AND LIMITATIONS

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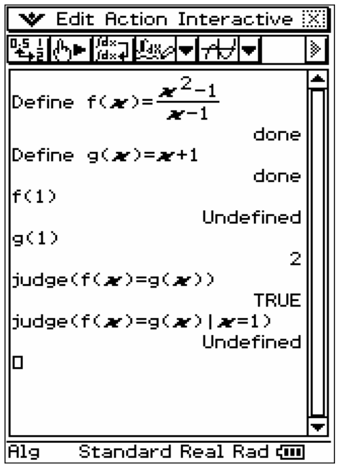
By using examples from upper secondary mathematics given for the Casio ClassPad tool, this presentation deals with determining equivalence of algebraic expressions, relating different representations, and making Computer Algebra Systems (CAS) techniques transparent and congruent with their paper-and-pencil versions. For each of these three topics, the presentation underlines its importance for CAS-based research, shows what best can be achieved with this tool at present, and summarizes limitations that should be addressed in a future version of the tool. In order to do mathematics with CAS in a better way, both CAS features and their use need to be improved.

INTRODUCTION

The use of CAS has been an important issue in secondary school mathematics for more than ten years (see www.lkl.ac.uk/research/came/). From the beginning of using such tools in the 1980s, there has been a development in improving their possibilities. By applying the perspective of working mathematically, this paper presents examples of possibilities and limitations of *Casio ClassPad* (www.classpad.org). These examples are given for three topics that are particularly relevant to CAS-based school mathematics. These topics are determining equivalence of algebraic expressions, relating different representations, and making CAS techniques transparent and congruent with their paper-and-pencil versions.



Screenshot 1

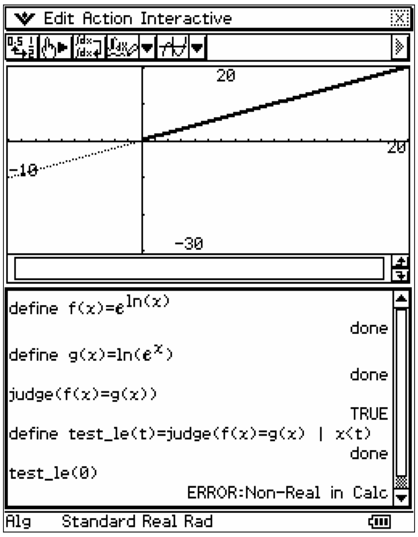
DETERMINING EQUIVALENCE OF ALGEBRAIC EXPRESSIONS

Topic importance

A large part of doing mathematics deals with transforming expressions to fit imposed requirements. Considering equivalent expressions is hence an important topic in mathematics teaching. Reasoning about equivalent algebraic expressions is an important research area not only in traditional mathematics education, but also in CAS-based mathematics education [1], where due to CAS limitations, students should skillfully use various CAS commands (e.g. the equality test and solve) to find out whether two expressions are equivalent.

Tool affordances

There is a ClassPad command named **judge** that is able to provide the answer to the question of equivalence in most cases. But, as shown on Screenshot 1, despite



Screenshot 2

a restriction of domain (with "|"command), the use of judge may be of little value with special subtle cases. The strength of command judge may be improved as presented on Screenshot 2. We find here that the user, perhaps believing **f(x)** and **g(x)** are not equivalent, applies function **test_le** (defined by him/her or other user) and gets message "ERROR: Non-Real in Calc".

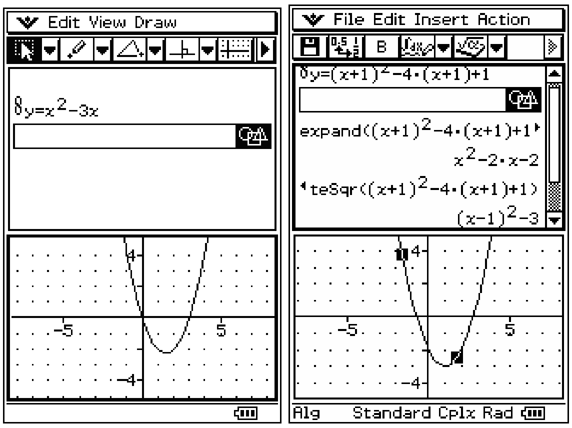
Issues to improve

According to [2], equivalence of expressions is not a decidable problem. In other words, due to theoretical reasons, equivalence of expressions cannot be verified in all cases even by ideal CAS. Because of that, subtle user-defined functions (created by teacher or able students) need to be used. This approach is not well supported by ClassPad at present as, for example, functions cannot be defined in several lines, or by using several commands in a line separated by ":". Also, there may be problems when using *if-then-else* statement implemented as **piecewise** function (see the appendix).

RELATING DIFFERENT REPRESENTATIONS

Topic importance

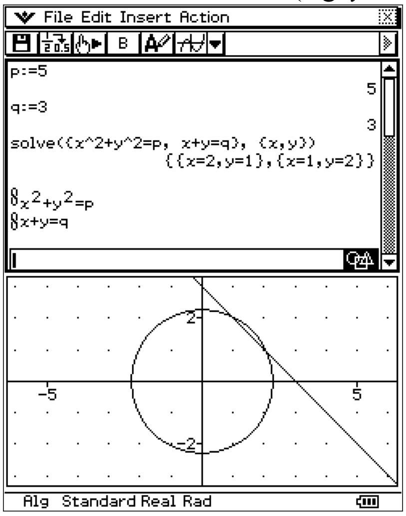
Using different representations of the same mathematical entity is one of the most important aspects of doing mathematics. A flexible dealing with different representations is usually seen as a step towards understanding of mathematical entities. As CAS are essentially representation tools, the question of creating, using and relating different representations is highly relevant to CAS-based mathematics education [3].



Screenshot 3

Tool affordances

Screenshot 3 present a ClassPad feature, *Geometry Link,* that links an algebraic representation of a function with its graphical representation in such a way that any change in one representation is reflected in the other. This feature is particularly useful in examining graphs of elementary functions under certain transformations (e.g. $y=x^2$ vs. $y=(x-1)^2-5$).



Screenshot 4

Screenshot 4 presents how two geometry links can be used in solving systems of equations, where solutions (2, 1) and (1, 2) can be recognized as the points of intersection of the two graphs.

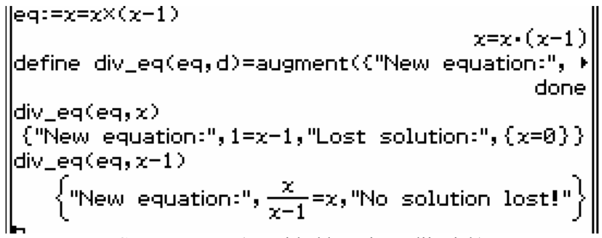
Issues to improve

The Geometry Link feature is limited, however. Indeed, if we change the values of parameters p and q (Screenshot 4), the solutions of a new system will be found, but the two graphs will not change. Also, a geometry link with a function on a restricted domain (e.g. $y=x^2 \mid x > 0$) will wrongly result in the graph of that function on its full domain. These two examples evidence that ClassPad applications (e.g. e-Activity and Geometry) need to be better linked in a future version of the tool.

MAKING CAS TECHNIQUES TRANSPARENT AND CONGRUENT WITH THEIR PAPER-AND-PENCIL VERSIONS

Topic importance

Paper-and-pencil techniques may not be reflected in their CAS versions (e.g. factorization of $x^9 - 1$). Also, CAS technique (i.e. the way in which CAS solves a class of problems) is not open to its user (hence the requirement: turn a back box of CAS technique into a white box or a grey one. (Term technique is used in the sense of the French school [4].). The success in doing mathematics with CAS is thus considerably constrained by the transparency of CAS technique as well as the congruence of that technique with its paper-and-pencil version [5]. Without reducing these constraints, CAS cannot be used in a functional, strategic and pedagogical way as properly required by [6].



Screenshot 5 (provided by Dj. Kadijevich)

Tool affordances

In order to have equation and inequality solving with CAS that is transparent and mirrors the usual paper-and-pencil work, several functions can be defined (by teacher or able students), explained to students, and used by students to help them be more aware of the solving processes. An example is given in Screenshot 5 (we divided both sides of the initial equation by x and then by $x-1$ and obtained proper answers).

Like the Casio FX2.0 CAS calculator, ClassPad also sometimes accepts syntax where parentheses are left un-closed (e.g. **solve**($x^3-1=0$ returns {x=1}). Some students like to use this time-saving, but counter-mathematical, feature.

Issues to improve

In order to improve shortcomings concerning issues of transparency and congruence, better CAS commands and appropriate user-defined functions are needed. These functions can be defined with ClassPad in a limited way as described in the end of the section on expressions equivalence. Also, some useful functions known to ClassPad (e.g. **element**({1, 2, 3}, 2) and **completeSqr**(x^2-4x)) cannot be found in the ClassPad manuals. Finally, some functions may still work in a strange way (e.g. **mode**({a, b, a}) is a, **mode**({true}) is TRUE, whereas **mode**({true, false, true}) is {TRUE, FALSE, TRUE}). All these make the work with user-defined functions a hard and somewhat frustrating job.

CLOSING REMARKS

Good user-defined functions are crucial to improving CAS. This requires CAS manufacturers to provide better conditions for the development of user-defined functions, taking into account critical issues given in this paper.

Contrary to paper-and-pencil mathematics, "defining a function is required before an expression such as $f(x)$ or $f(2)$ can be used." [4, p. 68]. Further CAS-based research may focus on the work with user-defined functions.

ACKNOWLEDGEMENT

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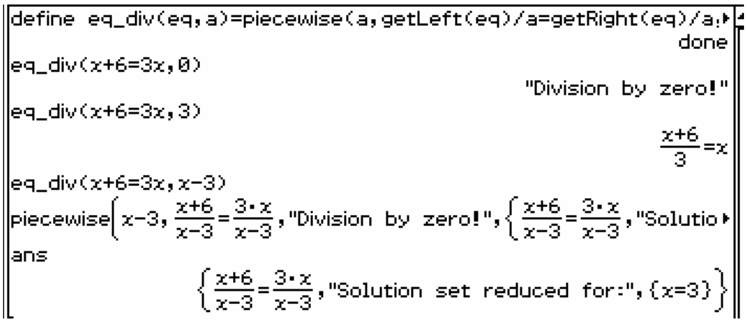
APPENDIX – USING PIECEWISE FUNCTION

Piecewise(0, 1, 2, 3) returns “2”, **piecewise**(2, 1, 2, 3) returns “1”, whereas **piecewise**(x, 1, 2, 3) return “3” although “3” should be returned in each of these three cases as the first argument of this if-then-else-error function is not a relation. By exploiting this behavior in a constructive way, define the following function:

```
define eq_div(eq,a)=piecewise(a, getLeft(eq)/a=getRight(eq)/a, "Division
by 0!", piecewise(judge(getLeft(eq)=getRight(eq) | solve(a)),
{getLeft(eq)/a=getRight(eq)/a, "Solution set reduced for:", solve(a)},
getLeft(eq)/a=getRight(eq)/a))
```

Screenshot 6 evidences that this function cannot be executed directly because Casio ClassPad returns **piecewise**(x–3, …), which is executed by using system variable **ans** containing this intermediate answer.



Screenshot 6