

TEACHING WITH A CAS-DG TOOL

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By considering a French experiment with Casyopée, this paper examines the theoretical frameworks underpinning its pedagogical plan, the first-adaptor teachers' experience with the tasks and tools, and the challenges of disseminating the tool to mid-adaptor teachers. It emphasizes conceptual development through inquiry into big ideas and professional development driven by sound principles.

INTRODUCTION

The ReMath Project (Lagrange, 2005, in [1]) undertook a significant challenge in developing Casyopée, an integrated CAS-DG tool that allows students to explore algebraic functions as models of geometric dependencies and co-variations. An innovative aspect of Casyopée is the ability to construct geometric figures in the DG component and export geometric dependencies into the CAS component. This feature allows students to view a linked geometric co-variation and algebraic function that models it. Focusing on the interaction of teacher with tool, I address three issues raised in Lagrange's paper: theoretical frameworks underpinning a pedagogical plan, first-adaptor teachers' experiences, and challenges of disseminating the tool to mid-adaptor teachers.

THEORETICAL FRAMEWORKS

The ReMath Project integrates four theoretical frameworks as a foundation for the project, providing a basis for task development and driving the pedagogical plan:

- Setting Notion (Douady, 1986, in [1]): supports the project outcome of switching between geometry and algebra settings and the need for a linked CAS-DG tool;
- Registers of Representations (Duval, 1993, in [1]): supports the goal to develop flexibility in changing registers underlying the concept of function;
- Instrumental Genesis [2]: underlies a pedagogical plan that addresses the teacher's/ student's knowledge of mathematics and the technological tool's capability for multiple representations; and
- Theory of Didactical Situations [3]: casts task design and implementation as a continuous interaction between a subject and milieu in didactical situations.

These theoretical frameworks provide a strong foundation for ReMath's pedagogical plan. The concept of associated function is introduced as a prerequisite concept, supporting the transition from geometric co-variance to algebraic model. If functions are to be used as models, then the concept of function must be well developed. In our work with college algebra students [4], we assessed conceptual understanding of function as the ability to shift between process and object interpretations of function and analytic, graphic, and numeric representations of

function in a Function Concept Matrix (Figure 1). Translation between some representations is easy for students, such as the numeric to graphic move that requires the lower level skill of plotting points. However, translating from numeric to analytic requires a more sophisticated understanding of modeling data. The need for understanding functions algebraically combined with the need to translate between geometric and algebraic conceptions of co-variation results in a complex task for students. In fact, some teachers might need explicit professional development to develop an adequately deep conceptual understanding to employ Casyopée's DG-CAS link effectively.

Interpretations	Representations			
	Verbal/Written	Numeric	Graphic	Analytic
Process				
Object				

Figure 1: Function Concept Matrix

The pedagogical plan addresses understanding of area relationships and prerequisite knowledge for geometric modeling, as well as an opportunity to explore the DG component of Casyopée. As with all powerful mathematical tools for dynamic exploration, there is a steep learning curve. Teachers and students need time to play and discover the capabilities of the tool as instrumental genesis evolves. In using Derive extensively with our college algebra students, we provided detailed directions concerning commands they were to use. Still, students seemed to be overwhelmed at times by the power and flexibility of the tool. I found the CAS and DG components of Casyopée, but not the interface between the two, to be fairly intuitive. Exporting between CAS and DG without significant direct instruction could be a point of confusion for both teachers and students. This raises a broader issue about the complexity and power of CAS and DG packages. While having a package like Derive or the Geometer's Sketchpad that provides an open workspace and numerous tools allows the teacher to address an almost endless variety of tasks, it often overwhelms both teacher and student. Only first-adaptors, using Lagrange's term, seem to make the effort to teach with such a tool without extensive professional development. How do we address the issue of powerful tools that are either used minimally (in terms of time or level of sophistication) by teachers? One approach is to develop micro-tools such as Java based applets. Micro-tools focus on one conceptual idea and provide only the features needed to address that idea. This reduces the confusion and time to impact in the classroom for the teacher and student. Casyopée is a powerful tool that may benefit from reducing some of its features to focus on the concept of geometric co-variations modeled algebraically.

FIRST-ADOPTER TEACHERS

The ReMath Project is to be commended for its efforts to study the impact of Casyopée and tasks developed around it on teacher adoption of the tool. Lagrange points out the pressing need to study the dissemination of technological tools through the stages of implementation, including:

- Development with first-adaptor teachers and experts,
- Proclamation by teacher educators as an innovative tool through preservice teacher courses or professional development for teachers in the field,
- Innovation in the classroom by expert teachers,
- Classroom implementation by a community of practitioners (mid-adaptors), and
- Use of the tool by most teachers.

I am always dismayed at the apprehension of preservice teachers and the dearth of implementation by classroom teachers concerning the use of CAS and DG. CAS, DG, and data analysis tools (such as spreadsheets and Fathom) are tools with great potential for improving student understanding, but in the U.S. they are greatly underutilized. Although the implementation barriers identified by Lagrange's – close to curriculum applications, time to implement technology, curriculum coverage issues, school organization issues, and professional development needs – are real problems, I believe that, until we move mathematics instruction from teaching for mastery of isolated skills and procedures to teaching for understanding, improving dissemination of technology tools will be very difficult. Mid-adopters and third-layer teachers, using Lagrange's terms, are pressured by high stakes testing and see these tools as add-ons or potential distractions.

The theoretical constructs addressing the dissemination issue are well taken. The boundary objects construct promotes mutual negotiation and meaning-construction as a norm, and should be a basis of good professional development. This leads to communal design of artifacts by researchers and teachers, as was done in the ReMath project. However, communal design is restrictive, impacting primarily the first-adopter teacher. While I agree that we must develop an understanding of how to engender the process of instrumental genesis, I maintain that to impact mid-adopter and third-layer teachers in the United States we must fundamentally change the expectations for what it means to learn mathematics.

Lagrange's two first-adopters had ownership in the project's work, so they were willing to overlook Casyopée's limitations. They had an intimate understanding of the intent and purpose of the tool, so they could more easily guide their students to desired outcomes while avoiding pitfalls. I venture that the first-adopters possessed an in-depth understanding of geometric co-variance and function as

model, either before the software development process or because of it. This reduced the complexity of the task for them, allowing them to focus on implementation of the tool in the classroom. For teachers to be successful implementers, they must have an in-depth understanding of the mathematical concept that is the focus of the task as well as experience with how the tool elicits this understanding from students.

DISSEMINATION CHALLENGES

As Lagrange noted, the six mid-adopter teachers had both experience with teaching with technology in the classroom and a positive attitude about the role of technology in teaching mathematics. They did not possess the first-adopters' knowledge of the development of Casyopée. The mid-adopters took six months to learn about the software before being deemed comfortable to implement the lesson. So they had an extended exposure to the tool. They also received mentoring from the expert teachers.

Despite this support, the mid-adopter teachers still displayed some traits of novice users. It appears they were more focused on the procedural and algorithmic aspects of the program than they were on the conceptual development aspects. When they implemented the tool in the classroom they focused on the CAS component and its ability to perform algorithms, ignoring the scenarios on modeling geometric dependency. The mid-adopters failed to comprehend, or at least target, the enduring concept that was the focus of developing the tool.

TWO ADDITIONAL ISSUES

The key question that arises for me from Lagrange's article is how the mathematics education community can move CAS and DG from use by a relatively small number of innovative first-adopter teachers to tools that are widely used. I believe that the primary barriers to further implementation of CAS and DG tools in mathematics classroom are institutional policy of teaching for knowledge of mathematical algorithms and procedures rather than teaching for mathematical understanding and problem solving, and ineffective professional development.

Until school administrators, teachers, parents, and students accept that teaching for understanding is the central tenant of quality mathematics education, then the application of CAS and DG in the classroom will be seen as extraneous. Current national and international high stakes testing with closed-form items, such as TIMSS, PISA (and NAEP, ACT and SAT in the U.S.), exacerbate the teaching for knowledge dilemma by assessing primarily knowledge and not understanding. Wiggins and McTighe [5] created a backward design process with the goal of engaging students in inquiry and uncovering big ideas of content. If the

mathematics education community can help teachers to value understanding developed through student inquiry into big ideas, then there will be a natural role for CAS and DG tools. I submit that the first-adopter teachers in Lagrange's project already valued inquiry and understanding, so they saw the importance of Casyopée. Moving mid-adopters and third-layer teachers to a pedagogical view of teaching for understanding is an essential criterion for improved dissemination.

Sustained content-based Professional Development (PD) is the second key to dissemination. The National Council of Supervisors of Mathematics [NCSM, 6] produced a summary of effective PD principles that call for school administrators, teachers, and PD providers to collaborate across three interconnected program components:

- **Diagnosing** the instructional problem to be solved. The instructional problem that supports the implementation of CAS and DG is moving from teaching for knowledge to teaching for understanding, which includes actively engaging students in open-ended explorations around enduring mathematical concepts, making CAS and DG tools an essential part of the curriculum, not an add-on.
- **Designing and Implementing** intervention strategies that are appropriate, including how they will be introduced. Research indicates that effects on teachers' classroom practice first appear after 30 clock hours of PD and increase through at least 80 hours. If Casyopée is to be integrated into the classroom, all participating teachers must receive extensive and sustained PD focused on mathematical concepts. The ReMath Project took strong actions in this area and still had teachers struggle to implement the tool.
- **Evaluation** as ascertaining what is working and looking back at what worked. Evaluation should take place on a two-tiered level, change in teacher practice and improvement in student learning. Pre-post data on student understanding of geometric co-variation and how Casyopée impacted conceptual development should be gathered. The peer mentoring aspect implemented by ReMath is recommended by NCSM, including observing lessons taught using Casyopée and experts modeling its use in the peer teacher's classroom.

CLOSING REMARKS

As Lagrange notes, encouraging second adopters and third-layer teachers is non-trivial work. Perhaps providing sustained effective PD and developing a focus on teaching for understanding for inquiry into big mathematical ideas, along with improved technology tools and tasks, will provide a mechanism for improved and extensive classroom implementation of CAS and DG.

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