FORUM 2/PRESENTATION
The use of CAS in teaching mathematics: Reflections on possibilities and problems of cooperation between theory and practice on the basis of a research and development project

Edith Schneider
Mathematical Didactics Department, University of Klagenfurt, A-9020, Austria
edith.schneider@uni-klu.ac.at

This paper describes a research and development project whose main focus is the use of a Computer Algebra System (CAS) in mathematics classrooms and the didactical possibilities linked with its use. The possibilities and problems of cooperation between science and practical work, between academic didactics and teachers are brought into view.

In the first section of the paper the central goals of the project are described; in the second and third sections the results of the project are presented, providing examples based on selected mathematical contents. Self-critical reflections on the project and the results are covered in the fourth section. Perspectives and consequences conclude the paper.

1. Goals of the project

Since November 1996, that is, for slightly more than 2 years, we have been working on a research and development project with the title "*Use of TI-92 in Teaching Mathematics*" at the Mathematical Didactics Department of the University of Klagenfurt (head of the project: W. Peschek).

Throughout this project, qualified expert, didactical and organizational consultation and support will be offered by us to mathematics teachers who would like to attempt to change their teaching (which has been "computerless" up to now) in a didactically sensible way into CAS-supported teaching. The extent and the focus of the support will change as demands and requests arise during the work on the project.

The target group of our project is mathematics teachers who are indeed interested in the use of CAS in mathematics classrooms and in the possibilities linked with the use of CAS, but who have not yet worked with CAS in their classrooms. These teachers would not like to make such a change alone, but would like to get (external) support. The main goals of the project are as follows.

Conception and development of a continual course of instruction for CAS-supported mathematics teaching.

Our goal is the development of a long-term didactical conception for using CAS in mathematics classrooms. At the end, we will be able to present continual teaching-conceptions which are didactically well thought-out and have been in practice at least for one grade. These will eventually be available, for more than one grade and include CAS-supported final examination. Based on these conceptions, appropriate teaching materials should also be developed within the project. That means, our intention is the formation of didactical conceptions for CAS-supported teaching of mathematics which is oriented on the curriculum. So, we are not interested in developing mock lessons of CAS-supported teaching for some selected mathematical topics, as is the case, for example, with the Austrian CAS-project concerning grammar schools (head of the project: H. Heugl - cf. Heugl, 1996). Within that project, complete lessons of some selected subjects of the curriculum of various grades, so-called observation windows, are made available to teachers for use in CAS-supported teaching. We will not confine ourselves to conceptions for mathematical topics whose treatment offers obvious possibilities for using CAS, rather we want to develop a continual instruction course for CAS-supported mathematics teaching. We will develop teaching conceptions for all topics covered in the curriculum of one or more grades. Thus, it is not intended just to take "mathematical highlights" and consider and investigate them from the point of view of the use of CAS.

Within our project, this goal has had effects on the choice of technology. Realizing it requires constant availability of a CAS for every student in or outside of the classroom, and so we decided to use the TI-92 as technological system.

Close cooperation between science and practice ("action research")

The development of teaching conceptions and teaching materials is to be done in close cooperation between the teachers and us, that is between science and practice. Thus, it is not the main point of our project to consider empirically a problem from "outside", to investigate it and find a (theoretical) solution which, consequently, is to be realized by the teachers participating at the project. On the contrary, it is that the participating teachers are to be "researchers" and "developers" by themselves and that they can independently come to solutions. They are consulted and supported in these efforts by didactics (the method of "action research").

For enabling such an intensive and permanent cooperation between teachers and scientists to take place, it is necessary to confine oneself to a few teachers. We started our project with two teachers at two different schools ("Handelsakademien"[5]) in 1997. Both of those teachers had showed interest in the use of new technologies in mathematics classrooms, but had had no previous experience at all in this matter. Both of those teachers would not have commenced with such fundamental alterations as the change to permanent use of CAS without (external) support.

---

[5] "Handelsakademien" are schools in which the curriculum is business oriented and which are completed with school leaving exams. Having completed these schools pupils are entitled to attend university.

Initially, the teachers had decided on the following model for collaboration: Both teachers would work together on the conception and development of teaching materials; we would act as consultants and observers of their work whenever the teachers requested our help or whenever we considered it to be helpful or constructive. But experience has shown that we would have difficulty attaining the desired results and success with this form of cooperation. On the one hand, our comments on the developed conceptions and teaching materials could only be made after the fact. This was not only very time consuming but also mentally burdening for all participants. On the other hand, due to the fact that the schools were approx. 100 km apart, there were difficulties in consulting and coordinating with each other about the individual chapters. The development of teaching materials became more and more individual work.

While conducting the developmental (preparation) work for the current school year, we tested a different cooperation model: the two teachers would work separately on the conception and development of their teaching materials, but will be constantly supported by one of us. In this way teaching materials for various subject matters are drafted and developed in parallel by the two "teams of two". Experience up till now has shown that, by proceeding in this way, the problems of coordination observed in the first model could have been eliminated or reduced. But the cooperation, contact and exchange of experiences between the two teachers continues to decrease.

2. CAS-supported mathematics teaching: Results obtained by the project

A lot of essential results, which have also been reported in other projects on the use of CAS in mathematics teaching, are also to be observed in our project.

Changes in social behavior and teaching forms

Clear changes in the social behavior and teaching forms can be recognized by observation of the lessons. The observation of the lessons took place for both teachers in classrooms which were not supported by CAS, as well as in the following school year in classrooms which were supported by CAS. The observations referred to the complete instruction course:

Study Groups (social behavior)

Approximately 90% of classroom time, which in a traditional classroom would have been conducted in plenum, was reduced to approx. 50% in CAS-supported classrooms. Approx. 50% of classroom time is spent with the students working independently (alone, or in small groups).

	without TI-92	with TI-92
1) plenum (teacher lecture, questioning-developing)	88.4%	53.0%
2) group work, partner work	0.0%	17.5%
3) individual work	10.7%	9.9%
4) combination of 2) and 3)	0.8%	19.2%

Dominance of Communication

The 80% of the classroom communication which had previously been determined and dominated by the teacher is divided into approx. 50% teacher dominated communication and approx. 30% student dominated communication in a TI-92 supported classroom.

	without TI-92	with TI-92
Teacher dominated communication	*79.3%*	*53.8%*
teacher lecture	9.9%	6.2%
questioning-developing classroom	41.7%	22.5%
answering teacher questions	27.7%	25.1%
Student dominated communication	*4.1%*	*31.8%*
student discussion	1.2%	11.8%
answering student questions	2.9%	20.0%
no verbal communication (silent work)	*9.9%*	*5.6%*

Active Participation of Students in the Lessons

While in the non-CAS-supported classroom many students actively participated in half of the lessons this is the case in three quarters of the lessons in the CAS-supported classroom.

The observation data presented indicate an average of the two teachers and it could be added that significant differences could be found between the two teachers.

	without TI-92	with TI-92
one student	2.1%	0.6%
a few students	46.7%	21.1%
many (the majority of) students	51.2%	78.3%

Increased experimental, heuristic treatment of subject matters

In the teachers' traditional lessons, experimental approaches to mathematical concepts could not be observed. An investigation of teaching materials from this time and discussions with the teachers about the conceptions of their lessons clearly showed that in most cases mathematical topics are presented by the teachers and then are practiced by the students on the basis of similar exercises. (The mathematical topics were almost extensively algorithms for problem solving.)

In the CAS-supported lessons, again and again we could see periods with experimental approaches or the first signs of heuristic problem solving. So, for example, the characteristics of exponential functions were investigated in an experimental way by examining the consequences of varying the parameters in logistic models of population growth. Or another example: the system of rules for the derivative of power functions were constructed in an experimental way.

Changes of forms of representation

In the traditional lessons of the two teachers, the consideration and solution of problems was done almost entirely on the algebraic level.

In the CAS-supported lessons, we did not see such a dominance of one form of representation. Forms of representations (graphical, algebraic, tabular) changed again and again; this happened with the introduction of mathematical concepts as well as with the application of concepts already known. So, for example, the students used recursive equations of functions as well as algebraic equations of functions when considering exponential functions; they also used graphical representations and tables. In the field of calculus it was not only the algebraic description of the difference quotient and the differential quotient which was emphasized, but also graphical representations and tables as far this is possible with regard to the limitation of computers for discrete mathematics.

Using modules ("Modularization")

In the CAS-supported lessons, the possibility of modularization offered by CAS was exploited (cf. Dörfler, 1993). The students operated with modules implemented in the CAS such as `solve`, `expand`, *d*, $\int$, drawing the graph of a function, creating tables of values of functions, and so on, as well as with modules defined by the students themselves. The latter modules are mainly rules, algebraic expressions stored as functions such as for example trigonometrical rules. This possibility is also especially useful for formulas in the field of financial mathematics (present value of annuity, accumulation of annuity, ...)

When using modules, it depended on the given situation, as well as on the given main emphasis and the given goals, as to whether the algorithms inside the modules were unknown to the students or whether they were made known to them and to what extent (cf. for example Drijvers, 1995; and the *Principle of Outsourcing* as a response to the problem of Black Boxes - cf. Peschek, 1998).

Increased application orientation

The teaching materials developed for use in CAS-supported lessons show an effort to create a link between mathematics and reality; these are used at the introduction of mathematical topics, in exercises and on tests. While the concentration in teaching up till now had been on inner-mathematical problems with high operative parts, in CAS-supported teaching application orientation has been more and more emphasized. So, for example, the growth of population has been modeled (including criticism of the model), the development of diseases has been simulated or possible functions of speed have been constructed for different racing circuits, or practice fields have been analyzed (projects in the field of financial mathematics).

In addition to these results of our project, we could recognize that the changes in the teachers' teaching have been taking place continually through the school course. Changes concerning the orientation of the topics and the goals, as well as the methods, could be observed continually for all topics in the curriculum. These changes can be documented by comparison of the teaching materials (exercise books, workbooks, tasks of tests) of the two teachers' previous ("pre-CAS") lessons with those of the CAS-supported lessons.

A continuity of changes could also be recognized in the field of social behavior and teaching forms. This can be inferred, as has already been described, by observing the lessons. During the observations of lessons, our interest has not been in the analysis of a few lessons but in efforts to extend the observations over the complete course.

In the following section, on the basis of one mathematical topic, I would like to demonstrate briefly how the teaching of the two teachers will change if the conceptions are developed in a consequent and continual manner.

3. A Case Study: Exponential and logarithmic functions

3.1 Exponential and logarithmic functions in the non-CAS-supported classroom

The topic exponential and logarithmic functions had been dealt by the two teachers in the previous (computerless) classroom in the following way:

1st lesson:	introduction of the exponential function: writing up the equation of the function, listing areas of application, special points of the exponential function, table, graph; introduction of the concept of exponential equation and solving elementary exponential equations
2nd lesson:	solving exponential equations
3rd lesson:	solving exponential equations
4th lesson:	practicing tasks for the test; solving exponential equations
5th lesson:	solving exponential equations; introduction of the concept of logarithm and logarithms as inversions of exponents, logarithmic function as inverse function of the exponential function; equation of the logarithmic function, graph
6th lesson:	calculating logarithms, rules of logarithm
7th lesson:	solving exponential and logarithmic equations
8th lesson:	solving exponential and logarithmic equations
9th lesson:	solving exponential and logarithmic equations

The main goals of these lessons as well as the orientation of the topics and the methodical aspects have been

- acquisition and reinforcement of *operative capacities and skills*: solving exponential and logarithmic equations is in the foreground. This is (almost) extirely done by means of inner-mathematical problems (exercises) of increasing complexity. Less than a fifth of the nine lessons are used for the introduction of the new topic and of the concepts of exponential and logarithmic functions.
- acquisition and reinforcement of capacities and skills to *identify certain mathematical structures* and to operate with these structures in an adequate way.
- support of qualifications as *consequent thinking in a stringent way, precision, persistence, concentration, discipline.*

- *exponential and logarithmic equations* and procedures (algorithms) for solving them. Functional aspects, essential characteristics, applications of exponential and logarithmic functions have no relevance.

- *algebraic representation*. Changes of different forms of representation do not exist.

3.2 Exponential and logarithmic functions in the CAS-supported classroom

In contrast to this, the consideration of exponential and logarithmic functions in a CAS-supported classroom is demonstrated in the following points. The presentation is done on the basis of the table of contents of a workbook. This workbook (68 pages) has been developed by the teachers and ourselves within the project.

1. *Linear growth and linear decline:* The topic is introduced by a model where the students already know the basic type of function.

2. *Exponential growth and exponential decline:* The introduction of the concept of exponential function is given using a non-mathematical problem (development of cell-culture). At first, it is aimed at a recursive description of exponential functions and at an emphasis on the constitutive characteristics (constant relative change) of exponential functions. From that an algebraic description is deduced which results in the definition of the concept of exponential function. This is completed by graphical representations. What then follows is a comparison of

   Linear and exponential models of growth and decline processes and

   Some problems concerning exponential models of growth and decline processes. The emphasis on these problems is on the description of non-mathematical situations by mathematical functions and on the interpretation of the results in the corresponding non-mathematical context.

3. *Characteristics of the exponential functions* and consequences of various parameters on the graph are to be investigated in an experimental, heuristic way.

4. *Logarithms and logarithmic functions* and the consideration of characteristics of logarithmic functions (again experimental activities)

5. *Natural exponential functions, natural logarithms*

6. *Further models of growth*: model criticism in which the limits of exponential models of growth and decline processes are shown using concrete examples (increase in population, plant growth) and further ”fine-tuned” growth models are presented

   model of retarded growth

   model of logistic growth (discrete and continuous model)

7. *Problems for repetition and reinforcement*

The main goals as well as the orientation of the topics and the methodical aspects of the CAS-supported lessons have been

- emphasis on the *aspects of representation and of interpretation of mathematics*: mathematical functions as descriptions of (non-mathematical) situations, facts (processes of growth and decline).

- *application orientation, closeness to reality, modeling* (including criticism of models)

- emphasis *on (adequate) concept formation* (constitutive characteristics, central characteristics, relations, limitation of concepts, ...).

- *outsourcing of the operative activities*, of the procedures (algorithms) to the computer (TI-92).
- *different forms of representation* (symbolic, graphical, tabular, numerical) and an adequate and efficient application of every form of representation.
- *independence in activities.*
- *functional aspects* and considerations of growth and decline processes.
- *inductive, genetic approach*: The concept of exponential function is at the end of a long process in which one specific (non-mathematical) context is always the topic of consideration and discussion.
- *experimental approaches* and heuristic methods.
- *recursive description* as one form of description that supports the consideration and investigation of the constitutive characteristic of exponential functions.

This comparison clearly shows the movement in the basic orientation of the conception of teaching. In the teachers' CAS-supported classrooms such changes or shifts can be observed continuously for each mathematical topic discussed.

4. Self-critical reflections on the project

What was able to be achieved by a project of this form? What are the potential and problems which can be identified?

Changes in mathematics teaching on the part of the project teachers

From a didactical point of view we are very satisfied with the course of the project and with the changes in teaching on the part of the two teachers. In the CAS-supported classrooms of both teachers we were able to achieve lasting changes in content, in educational goals, in methods, and in forms of social interaction. The fact that lasting changes on various didactic levels were observed for the entire course of instruction and not only for the individual curriculum of each school year, is a development we view as being extremely positive. Changes in the didactic focus were achieved for the entire course of instruction for teaching conceptions; an occurrence which is also clearly observable from the outside. The problem of discontinuity that can occur between the coverage of "mock contents" in the classroom and the rest of the course was avoided in our project.

There are also hints that the work on the project has had an influence on "regular classroom teaching", that is on instruction in the classrooms in grades not involved in the project also taught by both teachers. Changes in particular in the goals and in contents were recognizable.

Thus, we think that work on the project has had a positive influence on the conception and planning of both teachers' lessons and that lasting changes in the classroom on various didactic levels were achieved. What we did not examine, however, was how these changes affected the pupils and their ability to understand mathematical concepts.

Independent work on the part of the pupils

The increased independence of the pupils in the classroom was proven (through classroom observation) to cover mathematics activities. This means that in the activities in the classroom, the pupils were principally concerned with using the TI-92 with mathematics and not with the use of the TI-92 itself. It wasn't certain keyboard combinations which were in the foreground, but rather solving mathematical problems. This behavior was observed by us throughout the school year and for various mathematical topics; it was not a phenomenon only of isolated lessons on particular topics.

We must admit, however, that we have not gathered hard facts and figures on this matter; we have reached our conclusions based on our own subjective impressions in observing the classes. These impressions do not prove that pupils' performance has exceeded elementary problem-solving and has reached a higher mathematical level. We are not able to prove to what extent mathematics itself, the mathematics concepts and terms, were in the foreground for the pupils' activities. We also are not able to offer any statement backed by quantitative data as to how often the availability of workbooks took the ambitious mathematics performance out of the pupils' hands.

Summing up, the classroom observations regarding the independence of the pupils are subjectively extremely positive, although we are able to make only a few statements about the mathematical level of this independence.

CAS and its role in changing mathematics teaching

The changes observed in the teaching concepts and planning of both teachers with regard to goals, contents and methods is orientated for the most part on existing didactic concepts (cf. Schneider, 1998). There are concepts in mathematical didactics which have been developed years ago, but have been taken notice of in classroom teaching only slightly if at all up to now. This orientation on "old" didactic concepts clearly shows however, that most of the changes we observed would have been possible even without a computer algebra system being available in the classroom. CAS has only played an indirect role in the changes in the classroom: the role of a go-between for didactics and teaching, that is between didactic theory and school practice.

Only a few of the observed changes were made sensibly possible by the use of computer algebra systems in the classroom. The topics affected are, for example, in exponential functions when dealing with the recursive description of growth rates or with the experimental way of examining the effects of parameters on the behaviour of functions. These mathematics topics are offered again and again in the literature, in project descriptions and documentation, as examples for the use of a CAS in a mathematics classroom, and which become the contents of "observation windows" (cf. for example Aspetsberger, 1997; Connors, 1997).

Falling back on already existing didactic theories seems for us to be a completely sensible and adequate beginning. However, these "old" didactic concepts would have to be adjusted to the new situation, to the existence of CAS as a permanent teaching material. But as we see it, presently, a didactic theory for the use of CAS is still missing. Elements of such a didactic theory already exist, but these have not yet been formulated neither have they been didactically worked out, nor connected with each other. It is not clear as to in which form these could be used in teaching practice, where and in what way it would make sense to use them in a complete teaching concept.

Within the framework of our project we had intended to work out some keystones for a CAS didactic in co-operation with our project teachers. In close co-operation between didactic theory and school practice, elements of a CAS didactic were to be developed and tested. Unfortunately, this intention was not realizable, and the following reasons quite possibly could have played a role:

- The number of teachers participating in the project was too low, therefore not leading to a synergetic effect on the part of practice. Co-operation between the teachers was not marked and self-dynamics on the part of practice were completely lacking. The ratio of practitioners to academics was 1:1, leaving the practitioners too weakly represented.
- Under the general settings offered us by the practice and the pressure of daily teaching obligations it was not possible for us to develop fundamental didactic concepts together with the teachers, without already-existing first approaches. Having time as well as finding more, was lacking on the part of the teachers, in order to really be able to carry out such an important matter of concern.

  Perhaps with regard to developing project theory together we were slightly too ambitious in our intentions of having consciously chosen teachers who had had little experience with the use of CAS in the classroom and so had not taken a critical look at the practical teaching with, and the didactic possibilities, of CAS.
- It is not clear to us as to whether our teachers are in fact representative of teachers in Austria: Both teachers are certainly above average in their degree of open-mindedness towards and enthusiasm for innovations. Nevertheless, their behaviour shows a rather low level of didactic experience, which of course was not a favorable prerequisite for taking a deeper, more critical look at fundamental didactic questions. In addition to that, we were quite often having to clarify and to discuss mathematics-specific problems and questions which took up much time.

5. Perspectives and consequences

What we had planned for our project was, parallel to the conversion to CAS-supported teaching, to develop along with the teachers a theoretical foundation for a didactics in applying CAS. We were not able to carry this out for various reasons. We had to accept the fact that it was not possible for us to realize two matters of concern simultaneously, namely

1. getting teachers who had never had anything to do with CAS to use it in a didactically highly developed manner and
2. simultaneously developing the fundamental concept of a didactic of CAS-supported teaching in co-operation with those teachers who were to use it.

   (The latter also cannot be inferred from all of the CAS projects of which I have knowledge.)

But the insights emerging from our project build a valuable base for experience and starting point for further work. Through the project, it was possible for us to develop a sensibility as to what didactically could be practicable in the field of using CAS. What was confirmed and became concrete for us were elements of a didactic for using CAS, which had also been shown in the course of other projects, such as modularization, different forms of representation, and working in an experimental way. However, we were not able to gain any deep insight into these elements and their relations from our project. For this, it is necessary to go beyond the kind of work possible within the framework of this project to a theoretical level. We need a theoretical foundation for a didactic for CAS use which offers reinforcement as well as connections between these fundamental elements. We are currently working on the development of such a didactic foundation and corresponding theoretical concepts.

With such theoretical input available one could return to practice, to putting the concepts into concrete forms and testing them. This could take place by means of a further project or in the form of teacher training and further education courses.

References

Aspetsberger, K. (1997): Classroom Experiences with the TI-92 in Mathematics for 16 Year Old Students. The International Journal of Computer Algebra in Mathematics Education, 4(3), p. 303 - 311.

Connors, M. A. (1997): Exploring the Logistic Population Model. The International Journal of Computer Algebra in Mathematics Education, 4(1), p. 111 - 115.

Dörfler, W. (1993): Computer Use and Views of the Mind. In: Keitel, Ch./Ruthven, K. (Eds): Learning from Computers: Mathematics Education and Technology. NATO ASI Series F, Vol. 121, Springer, Berlin - Heidelberg - New York, p. 159 - 186.

Drijvers, P. (1995): White Box/Black Box revisited. International Derive Journal, 2(1), p. 3 - 14.

Heugl, H. (1996): Symbolic computation systems in the classroom. International Derive Journal, 3(1), p. 1 - 10.

Peschek, W. (1998): Mathematische Bildung meint auch Verzicht auf Wissen. In: Kadunz, G. et al (Eds): Mathematische Bildung und neue Technologien. Klagenfurter Beiträge zur Didaktik der Mathematik, B. G. Teubner, Stuttgart-Leipzig, p. 263 - 270.

Schneider, E. (1998): New Technology: A New Chance for ”Old” Didactic Ideas? In: Proceedings of the International Conference on the Teaching of Mathematics, John Wiley & Sons, New York, p. 263 – 266

FORUM 2/REACTION
A skeptic replies to Edith Schneider's "On using CAS in teaching mathematics"

Theodore Eisenberg
Department of Mathematics and Computer Science, Ben-Gurion University, Beer-Sheva, Israel
eisen@math.bgu.ac.il

I will react to Edith's paper by first presenting a short summary of what I consider to be its highlights; then I will make some general comments on the paper itself, and follow that by enlarging on the comments and my skepticism that her approach to introducing CAS into the classroom is unfeasible on even a moderate scale. I will close by speculating on the question: Where do we go from here?

1. A short summary

Edith Schneider and her colleagues are trying to get mathematics teachers interested in using CAS in their classrooms. Specifically they are interested in getting teachers to realize the strengths of the TI-92 and other such computers. But she and her team are not interested in writing a curriculum for the teachers themselves, or to even write model lessons for them; Edith wants the teachers to do that for themselves. The goal of her project is, as I understand it, not only to get the teachers to realize the strength and power of the TI-92, but to have the teachers start musing about how the computer can be used in *their* classrooms--and to have the teachers build and experiment with constructing appropriate modules by themselves. Edith's project is very much an individualistic one, with teachers constructing their methods of using the computer in their lessons, how often they should be used, to what purpose computers should be used, etc. This is what she means when she writes: that it is their goal for the teachers to have a didactic *conception* of using CAS--to see how it can be used in every corner of the curriculum that particular teacher must teach. So, she and her team work with the individual teacher to develop material specifically for them. In the long run, there might be tremendous overlap between the materials two different teachers develop about a particular topic, but the materials developed are tailor made by the teacher for that particular teacher's classroom.

According to the data Edith has collected, introducing the TI-92 has changed the atmosphere and climate of learning in the classroom. Whereas the teachers used to lecture and not emphasize group work, they now lecture less and emphasize group work; whereas the teacher used to dominate the discussions in the classroom with the students asking few questions, the teachers seem more open in their lessons and the students ask more questions than they did before the computer became part of their classroom's fixtures. The atmosphere in the classroom has changed--all for the better, with the role of the teacher being less the source of information of knowledge to one of being more of a collaborator in helping the students discover knowledge for themselves.

Admittedly only two teachers have heretofore voluntarily participated in the project, but from every aspect which has been scrutinized, the results are encouraging and form the rationale for continuing on with this method of effecting teacher change.

2. General comments

Many of the comments and criticisms I will now raise were mentioned by Edith herself, in the fourth section of her paper, but perhaps I will be able to discuss them from a slightly different stance.

The sum total of the number of teachers who have heretofore participated in this project is two! Moreover, they were volunteers. Now what does that say to you? To me, if a person volunteers for such a project he is a very special person indeed. He *wants* to change, and he has already convinced himself of that. So more than half of the battle is already won. Edith doesn't have to convince these individuals about the strengths of bringing computers into their classrooms, they have already convinced themselves that they want them. The impetus for change has been internal, not external, and that is important to note. True, Edith is providing them with the vehicle to effect this change, but the impetus for change is coming from the teachers themselves, not from Edith and her team; and herein lies the major fault of this study, as I see it.

A total of two teachers participated in this study--two--not two hundred. Why only two? Because I believe that there aren't two hundred teachers out there who are willing to do the work her project demands from them. Her project is built on the premise that teachers, themselves, want to change--and if they aren't already convinced of this, they most certainly will be convinced of it once they see the wonderful benefits of using CAS in the classroom are known to them. Colleagues, nothing could be further from the truth. Let me explain.

Edith's method to effect change is a good one--it works--and it will always work for the two or three teachers looking to change their ways. It doesn't matter if she wants them to introduce CAS into their classrooms or if she would be hawking some other line. Having the teachers sit down and think about what they are doing, and then mend what they are doing by developing their own materials is the correct methodology. So what is the problem? The problem is that most teachers do not want to change--they feel that they are doing a good enough job and if the students aren't learning, it is not because of their teaching and their classroom practices, but rather, because of the students themselves. And for the most part, the teachers are right. They present the material well enough- it is their job to present the material, and it is the students' job to learn it--that is the way the game is played.

I have yet to meet a teacher who says he is rotten teacher. Have you? Sure we will all admit to bad days, but on the whole we are pretty satisfied with our performance in he classroom--and we work very hard at achieving it. Most of us hardly have a spare moment to reflect on what we do in the classroom, let alone to develop a curriculum tailor-made by ourselves for ourselves. And this is second major flaw as I see it.

Teachers do not want to develop a curriculum for themselves. They want it done for them--and there is nothing wrong with that. Edith's comment: "...we are not interested in developing mock lessons of CAS..." starts them off down the wrong road from the very beginning. That is exactly what she should be developing--mock lessons. Her method of curriculum development links the materials developed to the teacher's knowledge--and at best it can't be stronger than the teacher's own knowledge base. (How can a teacher develop material in areas of which he himself has no knowledge? Even with Edith's model of having experts working with the teachers is limiting in this regard. Edith's overall goal of getting teachers to develop conceptions about change and their implementation, is limiting from the outset.) The usual model of curriculum development --with teams of subject matter experts and teachers working together developing model lessons-- works. History has shown us that. OK, she wants to have us use CAS--fine, but her method of getting us to adopt CAS will affect a minimum number of teachers at best--and in her case, she has shown us that the number is two.

3. Enlarging on the criticism

I am a computer skeptic. I am absolutely convinced that computers can affect our teaching--but only in the small, not in the large--computers are a tool, and they should not be elevated to being more than a tool. Let's look at history.

Computers have been around a long time already--and yet they are still on the periphery of the curriculum. Most university courses in mathematics that I am aware of have not made them central to the material--they are a tool and not much else. And they shouldn't be more than a tool. Why? Because mathematics is concerned with a way of thinking--computers can be used to enable us to progress in developing thinking skills, but for the most part, we have yet to exploit this aspect of them. And that is not because we haven't tried, we have tried. A tremendous number of people are working on how to use computers in the classroom, but the materials they are developing are not being picked up by those in the field. Why? Because confusion abounds with respect to our goals and how to achieve them.

Nothing is more evident about the confusion on how to use computers in the classroom than from the policy statements we get from the NCTM itself-- and the NCTM is supposed to be guiding light in this area. Everyone wants students to learn--but to learn what? And how does one accomplish this?

A curriculum must change, but are there any minimal skills anymore which are sacrosanct? A good part of the school curriculum is to help students cement-in skills; but the open question is : does this practice in mastering the grammar of algebra contribute in some way to understanding underlying concepts? E.g., does working with removing parenthesis in algebraic expressions or mastering the arithmetic of signed numbers, factoring, etc. aid to our understanding of the underlying concepts? And if so, to what extent? Have you ever had a student solve the equation x^2=5x by simply cancelling off an x from each side of the equation, or by rewriting the equation as x^2-5x =0 and then use the quadratic formula? Would hours of factoring stop students from using such an approach to equations like these? Or is it better that we let CAS solve such equations for us and that we move on deeper into curriculum? These are open questions, which must be answered before we can really integrate computers into the classroom.

OK, computers can do lots of drudgery work for us, and they can do it quickly and accurately. Furthermore, I can accept that students do not need to know how to work with logarithmic and trigonometric tables any more, that linear interpolation is obsolete and that there are many words in the mathematics vocabulary that are obsolete too, like pecks, bushels, and computing cube roots. I am willing to accept this, but when I see in the literature there is no longer a need for students to know long division (with a three or more digit divisor), or that multiplication algorithm is obsolete--that only the understanding of concepts is important, I get very upset. I just don't believe that knowing the underlying concept is enough--and that notion is the direct product of having computers in the classroom. Let me carry this further though a personal example.

My own students have been brought up with calculators in the classroom. Last semester I was teaching a course to pre-teachers, individuals who were literally only weeks away from being certified to teach, and who had done all sorts of mental gymnastics in other courses getting to where they were. For the most part they were good students. We got to a point in the lesson where we had to find the square root of a number. And immediately they reached for their calculators. I don't know why this bothered me, but it did--and I said: pretend that the square root key and the x^y key on your calculators don't work --what is the square root of the number? Well, as you can imagine the students balked, and when they finally understood that I wanted them to compute the square root of the number, all sorts of moans could be heard in the classroom. But the bottom line was: they didn't know how to compute a square root! I was in shock--teachers who couldn't compute a square root--that is scandalous--and that is exactly where we are going. Indeed, with some teachers, like those in my class, we are already there! OK, now what does this have to say about pushing CAS in the classrooms?

4. Where do we go from here?

It seems to me that many at this conference we are looking to CAS and other such programs as a panacea--and in some ways these programs are a panacea. They enable us to help many students get further into the curriculum more quickly than we have ever been able to do before. But there seems to be no consensus of knowing what we want our students to be able to do. This is not a plea to return to behaviorism, but rather to develop guidelines. I, for one, want students to be able to compute a square root--and to understand what they are doing. (And I am not talking about memorizing an algorithm. There are at least 7 different ways to find the square root of a number (with the aid of a calculator which has certain keys broken.) I want students to be able to construct some of these ways. It is getting them to think mathematically.) Edith Schneider and her team are years ahead of themselves, because they are pushing programs and procedures as though they know the answers to the questions on the periphery. Should students learn long division, factoring skills, curve sketching, etc. etc. And if so, to what extent? These questions must be answered. What do we gain by adopting CAS, and just as importantly, what do we lose?

I think that Edith's approach to introducing CAS into the classroom is years ahead of itself--and that too many fundamental questions need to be answered before adopting it. Until those questions have rational answers, I remain a skeptic about using computers in the classroom.