

# KEYNOTE 1

## CAS as a mentor for the apprentice mathematician

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### Abstract

The acceptance of calculator technology as an integral part of the mathematics curriculum in schools, colleges and universities is a continuing subject for discussion with the level and heat of the debate rising and falling periodically. It's always the same issues and questions that are asked at all levels of the curriculum. "When pupils use calculators they will not be able to do 'such and such'. Should we allow calculators in the classroom before pupils can do maths by hand? Should we allow calculators in the examination?"

The debate is now exercising the minds of teachers in higher education with most mathematics and engineering departments in the UK banning graphic calculators from examinations and thus discouraging their use during the teaching and learning process. The use of Computer Algebra Software (CAS) such as DERIVE, Maple and Mathematica has been accepted (grudgingly by many) because they are only available on computers, and access to computers is 'easy to control'. A calculator with a built in CAS such as the Texas TI-92 introduces a new dimension to the debate in higher education.

In this lecture I will argue that CAS, both as software on a computer and on hand-held technology, should be an integral part of the student mathematicians tool kit. It is important that students are encouraged to use 'appropriate technology appropriately'. There is a natural tension between the need for students to be able to do mathematics 'by hand' and to rely on technology. By reflecting on a module of mathematics in a Mathematics Degree programme I will demonstrate the need for students to turn to technology as a natural approach to unpacking the theory. At the same time we will discuss the concepts and skills needed to be able to use the technology effectively. The lecture will also look at the results of some research with school students using the TI-92 and consider their beliefs of mathematics learning when they use such technology.

### Overview of the Lecture

Within most universities in the UK (and I restrict myself to the UK as it is the higher education system that I have most knowledge) there is a dichotomy between those who approve of the use of CAS in teaching, learning and assessment and those who are against it. It is somewhat of a surprise to find engineering departments banning the use of technologies such as graphic calculators in assessment components when, for the engineer, mathematics is a tool to aid in problem solving and not a subject in its own right. It could be argued that within a Mathematics degree programme, students should be discouraged from using such technologies as we need mathematicians to be good at mental mathematics and technology reduces such skills. This is the dilemma that I find myself in on a day to day basis. Working

with the research community of mathematics education and in the area of research into the use of hand-held technologies and CAS in teaching, learning and assessment there is growing evidence that such technologies can help to develop understanding. However many of my colleagues who are research mathematicians discourage students from using CAS such as *DERIVE* and graphic calculators in their modules (and yet these same colleagues use CAS such as Mathematica in their research!). So what does a student make of the situation where they are encouraged to use contemporary technology in only a few modules?

Of course it is not true that I do not struggle with the same issues. I am a CAS enthusiast and a teacher on a final year undergraduate module within the Mathematics degree programme. I often feel pulled towards the view that 'students should be able to do this or that using pen and paper techniques'. Inevitably there is the feeling that if I can do it 'by hand' then the students should also!

The evidence to support the use of CAS in teaching and learning began with the early work of Heid (1988) and Palmiter (1991) and has been replicated by several researchers since. They concluded that students using computer algebra software had a deeper conceptual understanding of concepts of calculus without losing traditional skills. Mayes (1996) gives a comprehensive review of research in the use of computer algebra software and concluded:

*There is compelling evidence that fully integrating a computer algebra software into the curriculum as a cognitive tool within the constructivist perspective can have positive effects on mathematics learning.*

*However, if CAS is appended to the curriculum, or used primarily as a computational aid, then learning may not be as dramatically affected.*

This last point is important because I believe that one of the challenges facing mathematics educators who support the use of CAS is how to fully embed the software into programmes of study. Much of the research work has involved the use of CAS on a computer in a laboratory activity. Although the activities have often been integrated into the courseware the technology itself has not been fully integrated in the sense of homework tasks etc. The inclusion of computer algebra software into hand-held technology such as the TI-92/89 changes the availability of such software in teaching and learning. No longer do students have to 'leave the computer behind in school or college'. We can now fully integrate the software as a cognitive tool to develop concepts and skills in mathematics. For the lecturer, using a TI-89 with an overhead projector in an ordinary classroom is so much easier than a PC. Furthermore it is easily integrated into the presentation as a 'natural tool' so that the apprentice mathematician recognises the value of CAS as an integral part of their toolkit.

I will briefly illustrate the use of CAS in a final year module on non-linear systems through some examples. The module has four parts: first order continuous autonomous systems, second order continuous autonomous systems, discrete systems and asymptotic methods of solving non-linear differential equations. I use the TI-89 and its computer algebra facilities throughout the course for drawing direction fields, exploring particular solutions, doing algebraic manipulations, as well as familiar graphing calculator activities. Students are encouraged to work with similar technology if they choose to buy it or use *DERIVE* in the computer rooms.

As an example consider the problem of finding periodic cycles for the quadratic discrete system:

$$x_{n+1} = x_n^2 - c$$

as  $c$  varies.

The quadratic function  $f(x) = x^2 - c$  is called the mapping function and the periodic cycles of period  $k$  are defined by

$$f^k(x) = x \text{ where } f^2(x) = f(f(x)) \text{ etc.}$$

An important property of the theory is that if there is a period 2 cycle and we are searching for a period 4 cycle then  $(f^2(x) - x)$  will divide into  $(f^4(x) - x)$ . The TI-89 screen dumps below illustrate these ideas for a choice of  $c = 1.3$ .

The function  $f(x)$  is defined and  $f^2(x)$  and  $f^4(x)$  are evaluated.

```

F1- F2- F3- F4- F5- F6-
Tools Algebra Calc Other Pr3mID Clean Up
Define f(x)=x^2-1.3 Done
f(f(x)) x^4-2.6x^2+.39
f(f(f(f(x))))
x^16-10.4x^14+42.12x^12
f(f(f(f(x))))
MAIN RAD AUTO FUNC 3/30
  
```

When we divide  $(f^2(x) - x)$  by  $(f(x) - x)$  we get the quadratic  $x^2 + x - 0.3$  which has roots 0.2416 and  $-1.242$ . These form the period 2 cycle.

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F1- F2- F3- F4- F5- F6-
Tools Algebra Calc Other Pr3mID Clean Up
Define f(x)=x^2-1.3 Done
f(f(x))-x x^2+x-.3
f(x)-x
solve(x^2+x-.3=0,x)
x=.2416 or x=-1.242
solve(x^2+x-.3=0,x)
MAIN RAD AUTO FUNC 3/30
  
```

When we divide  $(f^4(x) - x)$  by  $(f^2(x) - x)$  we get a polynomial of degree 12 which has roots 0.389, 0.0194,  $-1.149$ ,  $-1.3$ .

These form the period 4 cycle.

```

F1- F2- F3- F4- F5- F6-
Tools Algebra Calc Other Pr3mID Clean Up
f(f(x))-x x^2+x-.3
f(x)-x
expand(f(f(f(f(x))))-x)
x^12-7.8x^10+x^9+21.45
...((f(x))) - x) / ((f(f(x))) - x)
MAIN RAD AUTO FUNC 3/30
  
```

```

F1- F2- F3- F4- F5- F6-
Tools Algebra Calc Other Pr3mID Clean Up
expand(f(f(f(f(x))))-x)
x^12-7.8x^10+x^9+21.45
solve(x^12-7.8x^10+x^9+21.45=0,x)
x=.389 or x=.0194 or x=-1.149 or x=-1.3
Solve(x^12-7.8x^10+x^9+21.45=0,x)
MAIN RAD AUTO FUNC 4/30
  
```

What do students need to know to be able to follow these activities?

- an understanding of the function notation;
- the meaning of  $f^k(x)$ ;
- division of polynomials;
- idea of polynomial factors;
- roots of polynomial equations;
- algebraic manipulation.

How much of these activities need to have been carried out by hand before we use the technology? This question is a constant dilemma to me. I expect the students to be able to expand  $f(f(x))$  and divide  $\frac{f(f(x)) - x}{f(x) - x}$  by hand and to be able to demonstrate it in

examination conditions. Is this the traditional applied mathematician's view? What extra insight do students get by doing these things by hand?

But there is an equally more important issue to be considered. I have used  $c = 1.3$  in the example above. As part of the course style of whole class interactive teaching the students explore most of the values of  $c$  themselves. (For a discussion of the use of the pedagogical approach to the module see Berry and Sharp (1999).) The students are encouraged to use CAS during and after formal sessions to develop the theory within the course.

How familiar should students be with technology before using it to learn new ideas?

In mathematical modelling courses there is a rule of thumb that students should use 'one year old mathematics' when developing new models or modelling skills; similarly when using models to develop new mathematics (e.g. in engineering and science courses the mathematical topics are often introduced through applications) the models should be 'at least one year old'. What this 'one year old' means is that students should be familiar with the topic so that they are learning one new idea not two i.e. new models and new mathematics. I propose that the 'one year rule of thumb' applies to the use of technology and CAS for the apprentice mathematician. The implication of this is that if we want to use CAS to help in the teaching and learning of mathematics in higher education then their use should begin with familiar mathematics in upper secondary school! Perhaps more realistically in order to embed technology and CAS as a routine learning tool in higher education we should introduce it into the first year using upper secondary school mathematics.

Now I turn to the views of some of my colleagues. I want students to feel confident that CAS has an important role in their learning and in doing mathematics. But the message is a mixed one from my colleagues. Here are two statements from some recent discussions:

- ***Using a graphics calculator is cheating;***

several reasons are put forward for this statement: 'students should be able to do the same mathematics by hand that we can' – is this reasonable for all students? 'students can type up their lecture notes into a graphic calculator and then reproduce them in the examinations' – this is a possibility but I would argue that examinations should be about trying to test student understanding not repetition of notes but it means some thought being given to the assessment tasks;

- ***CAS is acceptable in the final year when some students are moving towards a research agenda:***

this is a popular comment in that the first two years of a degree programme is about gaining traditional knowledge and skills and then we can do the 'real' mathematics in the final year; but what about the 'one year rule of thumb'?

There is a surprising snobbery among my colleagues about CAS. A few of us have been using *DERIVE* for almost ten years now with engineering and mathematics students and more recently graphic calculators with CAS. This has been frowned upon and at the last revalidation of our degree programme (late 1996) there was some attempt to ban the use of CAS and graphic calculators. But recently one of the 'respected' research mathematicians has proposed the introduction of Maple in all years of our programme. For some reason this is acceptable! Yet the use of Maple will do no more than *DERIVE*!

I conclude this presentation with some questions in the emerging research agenda of the computer CAS/hand-held technology CAS duo. How can we and at what stage should we embed these technologies as natural tools in teaching and learning? What are the important differences in the duo that need to be identified and explored? What are our apprentice mathematics beliefs associated with teaching and learning with CAS – are they being cheated from being 'real mathematicians'?

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# KEYNOTE 2

## Rethinking abstraction in the light of digital technologies

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I guess I have been invited to this conference because the organisers believe that there are lessons on the introduction of digital technologies which are not age-specific, or particularly the preserve of computer algebra systems. I can think of several; in fact as I write this, I am hard pressed to name a research finding of any depth which is entirely specific to a particular age-group or ‘level’ of mathematical experience. It is, of course, in the nature of research with new technologies that the specificities of software intrude in very specific ways into the kinds of activities that people undertake, both from an epistemological point of view (the kinds of knowledge they engage with) or pedagogical (how they are encouraged to engage with that knowledge). But even here, there are broad-brush findings that apply, at least to the class of software that might be labelled ‘expressive’ in one or another way. And perhaps the most pervasive, and seemingly obvious finding is that learners need to have something they want to express in order genuinely to take advantage of the opportunities afforded by expressive media.

### Why do people study mathematics?

I’m going to base my remarks, inevitably, on experience of working in the UK — actually only England. I know this is dangerous, as the recent history of university (and school) education in my country has been far from glorious, and the trajectory seems mostly depressing: the details would be unseemly to display at an international gathering. One of the purposes of raising these issues at the seminar is to discuss the extent to which this situation is echoed elsewhere.

In our recent book, *Windows on Mathematical Meanings: Learning Cultures and Computers*, Celia Hoyles and I wrote the following paragraph:

The role of mathematics in underpinning social and economic life stretches back to the dawn of the industrial revolution and beyond. Every aspect of modern society is infused with the congealed mathematical labour of mathematicians, computer scientists, engineers and so on. Yet at the same time, this mathematics is increasingly invisible to those who merely share in, rather than construct, the artifacts of the culture. It is mathematics which lies dormant inside the chips of vacuum cleaners, the warheads of missiles, and the graphical displays of news broadcasts. Even the simple exchange of goods and commodities, once relatively amenable to mathematisation, has been overwhelmed by the workings of global

markets which are dominated by invisible mathematical forces which are increasingly out of control. (Noss and Hoyles 1996).

The problem of invisibility is nicely summed up by a recent report (1998) *Mathematics in Industry* published by the Society for Industrial and Applied Mathematics, who having surveyed mathematicians working in industry came to the conclusion that ‘Mathematics is alive and well, but living under different names.’

Of course this adds momentum to the idea that mathematics is an optional extra, even for avid consumers and users of mathematical ideas, and in Anglo-Saxon cultures at least, has led to an institutionalisation of this idea in the mixing of mathematics with almost any subject at University (the ‘modular’ course system has encouraged this still further) and the proliferation of books with titles like *Statistics without Maths* (Dancey C. & Reidy J, Prentice-Hall 1999).

The disjuncture between mathematics as a component of culture, and mathematics as a tool for society’s use, threatens to turn on mathematics itself, as more and more young people become alienated from what they see as a dehumanised field of study. In the school arena — and perhaps increasingly beyond it, this disparity feeds back strongly to those whose profession it is to help children learn mathematics; in some mathematical circles at least, there is certainly a crisis of identity in which lecturers as much as students are not altogether sure of their roles or even of the kinds of knowledge it is their duty to teach.

Unsurprisingly, therefore, young people are coming to Universities to study mathematics whose aims and rationales are at variance with the expectations of those who teach them. To make my point, let me start with a quote from a student, one of a number whom we have interviewed in a small-scale project to investigate the transition from school to university mathematics. Our student is studying at one of the country’s leading research universities, a university whose entry criteria are superceded only by two or three in the UK. We should assume then, that he is atypical of students in general, but quite typical of the very best and mathematically committed of his generation. We asked him how he was getting on his new course, and received the disarmingly frank response:

*Student:* I actually don’t see the point of proofs to be honest. He can just tell us yeah, a convergent sequence has a sub-sequence that also converges or whatever and that would be it, but he needs to go and prove it for some weird reason [...] it takes up all our time

*Interviewer:* So you’d rather be doing the of real working out of problems?

*Student:* Yeah, giving us examples on those theorems so that we could be able to do them.

There are two ways to interpret this observation. One is to lament that the student has no conception of what mathematics is about, that proof is absent from school mathematics and that he has no understanding of its central role, etc. But there is another, not necessarily exclusive, explanation. It is that the student has realised that the theorem itself is important because of its application (and its role in proving other theorems, solving other problems). It is this application which he is finding hard. And he doesn’t see why understanding the proof of the theorem helps with *his* difficulty in solving problems.

Of course, nothing has really changed. Generations of maths students have sat through proofs of theorems, some of which they believed they understood, only to find that they were unable to use these theorems in the solution of problems or examination questions. But what may be

different here, is that the student is unable to see why the proof might be relevant at all – that it is actually ‘weird’ to prove theorems in the first place.

We have a sample of one, so we do not want to posit any overblown theories. But in one respect, there is a basis for understanding the student’s perspective. Later in the interview, he says:

I don’t see how maths could help with management at all or public services. Maybe teaching maths, yeah, but it won’t help teaching maths at secondary level, because it’s so different...

If the student is studying mathematics to enter public services, then the most he wants is a good degree. And the path to a good degree is the solution of problems. He believes proofs are ‘weird’ because they simply don’t help him to do that (of course, we might argue that being able to prove a central theorem helps understand the place of that theorem in the scheme of mathematics, and that may be an essential part of using the theorem in its applications — the student doesn’t see it that way).

It is no use mourning the passing of generations of students who were in love with mathematical ideas for their own sake. My guess is that the UK (and other Anglo-Saxon cultures) is leading the way in this respect. I am not happy about it. But it won’t go away. And, for a host of reasons that are well outside the scope of this abstract, I suspect that the tendency will become widespread internationally, if it isn’t already.

One ‘solution’ to this malaise, is simply to damp down the abstraction of the mathematical enterprise. International readers may be surprised (or shocked) at the extent to which, in the UK, we have modified mathematics courses in recognition of the changing demographic trends of student intake. For example, Cambridge — together with almost all Universities (but not, interestingly enough, the one at which our student was studying) — has postponed the introduction of real analysis into year 2, as students — among the best in the country — do not arrive ready to make the transition to thinking abstractly (see Kahn and Hoyles 1997).

Perhaps these changes are only a recognition of the changing roles of mathematics? Perhaps Universities are right to recognise just how abstract mathematics is (we encountered in our project a ubiquitous belief among tutors that most students reached their ‘ceiling’ rather soon, and that some had reached it before entry), and that is right and proper to wrap mathematics up safely, leaving only the innocuous and ‘concrete’ bits showing through for most consumers to use:

The public wants the concrete; it does not like the abstract, and mathematics is surely one of the most abstract of the human construction.

Boos-Bavnbek and Davis, (quoted in Kahane and Chouchan 1994, p. 4).

The danger is that in the process of trying to make mathematics more learnable, it becomes ever more separated from sources of potential meanings; abstraction comes increasingly to appear synonymous with separation and dehumanisation; teachers and students alike have difficulty connecting the meanings of their lived-in cultures with new meanings from outside that culture; and the educational system stands ready to disconnect innovation from any sources of genuine change. And it is here, perhaps, that technology may play some constructive role. Put bluntly, how can we reconnect mathematics with the lived cultures of those who want to learn it, in ways which do not dilute out of recognition the essence of what it means to think and learn mathematically?

## Reconnecting mathematics and culture

The kind of thing I have in mind was illustrated to me most vividly by the collaborative engagement I have with the METRIC project at Imperial College London, and in particular the work of Phillip Kent within it<sup>6</sup>. Kent's work is designed for a final-year undergraduate mathematics course on "Dynamics" (a body of mathematical theory covering classical mechanics, as well as the recently grown-up fields of nonlinear dynamics and "chaos theory"). As Kent puts it:

This is a long-established course which previously has had no compulsory computer work, though typically a few interested students each year will do an optional computer-based project. Dynamics should be a natural arena for computer-based explorations; indeed, the growth of nonlinear dynamics and chaos theory over the past 30 years could only have taken place given the central role of computational experiment and visualisation. However, computing assignments are not popular with third-year students, because they know from past experience that these consume a lot of effort for little credit. One of the issues, then, was to find ways to negotiate this new presence in the course: how to develop ways of allocating credit to it whilst at the same time trying to do something new and interesting in the way of computer-based activity. (Kent, P. MA Dissertation, in preparation).

So the challenge was how to design activities which afforded exploration of the properties of the solutions to the ordinary differential equations which appear in dynamical systems, and to investigate central mathematical ideas. Studying these in detail is an abstract undertaking by any definition, and these students were certainly capable of such a task. But it is relevant that most of the systems which students encounter in the course are soluble *without* the sophisticated methods of advanced dynamics they are being taught — because the problems that they need to do for coursework and examination must be doable within the limit of a few pages of pencil and paper algebra. This situation, which I suppose is typical of many advanced applied mathematics courses, reinforces precisely the 'who cares' attitude we have already met. How, then, should students be given the chance to explore real systems, in manageable (and assessable) ways?

Kent's answer was to ask students to produce a "portfolio" of mathematical images. Kent says of these images, that they "can be compared with the idea of "curve sketching" in school mathematics, where a single-variable function is drawn so as to highlight its characteristic properties (roots, turning points, asymptotes, etc)."

I think this solution points to two important issues — one technical/pedagogical, and the other theoretical. The idea of a portfolio is, I think particularly apt. It fits more naturally with art, literature or photography, where the student is able to display a record of attempts at expressing the state of his or her understanding. This is particularly difficult in mathematics, as it is so often the case that students simply cannot get started in solving a problem, or just do not see how their knowledge can be coordinated to point towards a solution. I am attracted to the idea of a portfolio, because it speaks of knowledge in construction, expressing what *is* rather than what is *not* feasible for the student.

But there is a second, theoretical issue which I think Kent's idea illustrates. It is to do with abstraction. Building a *Mathematica* model is, in this respect, quite unlike a photograph or a painting. It is impossible to produce an image without carefully describing that image

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<sup>6</sup> I would like to thank Phillip Kent for his collaboration, and for his comments on a draft of this outline.

abstractly. Abstractly in some restricted sense, of course, as *Mathematica* code — however arcane — is not identical to the standard mathematical representation. And there's the rub: by asking students to present a portfolio, he is asking them to think abstractly about the images, in ways which are tied rather tightly to the particular representational system involved (in this case, *Mathematica*). We have no evidence that students who can construct formally representations of particular images can necessarily solve problems which involve working with abstract representations of them. In a strict sense, one might almost argue that *Mathematica* offers students the chance to escape abstraction altogether.

Nonlinear problems cannot, in general, be solved in explicit formulae, so the images provide manageable representations of numerical data. But they provide more, in that — provided one knows how to 'read' them — they afford powerful ways of thinking about the formal mathematics underpinning them. Of course, this is by no means automatic, and we have, regrettably, no data yet which will confirm or deny the hypothesis that students gain insight by the construction of the portfolios. This achievement would be, in any case, highly susceptible to changes in the initial conditions — what kinds of activities, what kinds of tool, what kinds of settings? Design is crucial here, as the tools we choose shape the expressiveness of the system (see, for example, Kent & Noss, in press; Noss, in press), and ultimately, the nature of the understandings that students construct.

We want students to think abstractly, but we simultaneously want them to feel that the ideas they encounter are, for them, concrete (Wilensky 1991, has written eloquently on the idea that abstractness is a relation between person and knowledge, not a property of knowledge alone). We would like students to be able to express themselves in some abstract or quasi-abstract language (like some version of CAS code), which may fall short of fully-fledged mathematical abstraction, but which *does* allow them to express ideas that make sense.

I conclude this outline with a research agenda which I will boil down to a statement and a question. Digital technologies afford new structurings of knowledge across old boundaries. Will this not mean that we will ultimately have to rethink what we mean by abstraction itself?

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# KEYNOTE 3

## Computer algebra in mathematical research

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### Abstract

Computer algebra systems, like Mathematica and Maple have become important tools for the working mathematician. I used Mathematica extensively in the study of certain problems related to Probability and to Combinatorics. An example of one such case was the discovery of a strange probabilistic phenomenon for “random Young tableaux”. That phenomenon was observed while making "Mathematica" calculations with these tableaux. That phenomenon is equivalent to a non trivial binomial-coefficients identity. The first proof of that identity was a computer proof by D. Zeilberger, using Maple. A “human” proof was found only about a year later.

Another way in which Mathematica was extremely helpful was in the verification of combinatorial identities. By applying rather “heavy” tools from Probability and from Group Representations Theory, we arrived at some explicit and very complicated infinite sum identities. Trying to write down explicitly such an identity, it is nearly impossible to avoid at first some human errors. Testing the written identity on Mathematica tells one if the identity indeed holds. If not — which is usually the case at the first trial — then one looks for, and usually finds, some computational error(s). It usually took 3-4 such steps before all errors were eliminated. Mathematica proved to be an indispensable tool in that research.