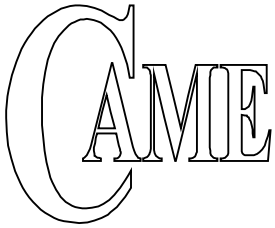


Summary of computer algebra activities in eight countries

In July 1999, one delegate from each of the countries represented at the Weizmann Institute CAME meeting was asked to provide a summary/overview of CAS projects (curriculum or research projects) going on in their country. The following pages are the result.

[http://metric.ma.ic.ac.uk/came/events/weizmann]

Summary of computer algebra projects in Australia — by Margaret Kendal et al

Australian educators are interested in using CAS technology in the classroom but it is not permitted during examinations. Calculators with CAS are still expensive relative to graphical calculators which are being used quite widely. In Victoria, students were permitted to use graphical calculators in their VCE examinations for the first time in 1998.

1. A selection of current research

   Barry Kissane, Murdoch University, W.A.

   Current CAS research: Algebra in secondary schools with a calculator

   http://wwwstaff.murdoch.edu.au/~kissane/ kissane@central.murdoch.edu.au

   Peter Galbraith & Mike Pemberton, University of Queensland, Queensland

   Current research: Attitude and performance among engineering students using Maple software

   Reference: Galbraith, P. (1998) I Keep Six Honest Serving Men: Teaching Mathematics in New Times, Conference Proceedings of MERGA 21 (Goldcoast), 16-28.

   edpgalbra@mailbox.uq.edu.au

2. CAS research reports to be presented at Mathematics Education Research Group of Australasia. MERGA 22, 4-6th July 1999, Adelaide (other CAS related research is likely to be presented):

   - Learning calculus with supercalculators (McCrae, Asp & Kendal, 1999)
   - Computer algebra systems facilitate positive learning strategies. (Pierce, 1999)

3. University of Melbourne Technology and Mathematics Education (TAME)

   The TAME CAS group (university staff and practicing teachers in Melbourne) meets regularly to examine the role and impact of CAS in secondary school mathematics curriculum, and carries out CAS-related research, designing and trialing CAS active classroom tasks and units of work, and teacher support.

   Recent research projects include:

   McCrae (1996): Impact of CAS on senior school assessment;

   Tynan and Asp (1998): Year 9 and 10 school algebra

   http://www.edfac.unimelb.edu.au/DSME/TAME

4. Current Ph.D. research projects supervised by Professor Kaye Stacey, University of Melbourne.

   Algebraic insight and CAS. Involves first year Ballarat University students using DERIVE. Robyn Pierce, r.pierce@ballarat.edu.au

   Impact of introductory differential calculus with CAS on conceptual understanding. Secondary school students using the TI-92, teacher related influences also monitored.

Margaret Kendal, kendal@bacchusmarsh.net.au

5. Curriculum development

David Leigh-Lancaster, Board of Studies, Manager Mathematics Key Learning Area, Victoria

David has an interest in curriculum development and was involved in the development of Mathematically Able Software for Schools (MASS) as part of the Association of Australian Mathematics Teachers AAMT LUDDITE Project

http://www.ash.org.au/teachers/aamtprof/luddite.html

david.leigh-lancaster@dse.vic.gov.au

Reference: Leigh-Lancaster, D.(1996). *Mathematica* at the Secondary – Tertiary Interface. Australian Senior Mathematics Journal, 10(2), 23-32.

Computer Algebra Projects in Austria

Werner Peschek, Department of Mathematics Didactics, University of Klagenfurt

The first teaching experiments in Austria using a CAS were initiated in 1984 (K. Aspetsberger, Linz) and in 1990 Austria was the very first country to purchase a DERIVE general license for all its high schools. Even so, this measure did not lead to a rapid wide-spread use of CAS in mathematics teaching in Austrian schools; in the long run it did lead, however, to a series of interesting initiatives on the part of individual teachers or smaller groups of teachers. For the most part, such individual initiatives have not been documented. Sometimes, these activities will have been presented and discussed at teacher conventions for further education, sometimes teachers are reporting on their ideas and experiences at CAS or didactic conferences.

Individual initiatives such as these provided the basis for three large CAS Projects:

CAS I: school year 1993/94, head of project: H. Heugl (Vienna)

19 high schools with 32 teachers, 38 classes consisting of 718 pupils (in grades 7 - 12) took part in this teaching project using DERIVE (with a PC) in the mathematics classroom. Further information (in German) can be found at:
http://www.asn-linz.ac.at/schule/derive/d.htm

CAS II: school year 1997/98, head of project: H. Heugl (Vienna)

This teaching project dealt with the use of TI-92 in the mathematics classroom. 71 classes with approx. 1700 pupils in grades 7 - 11 took part in this project.

What was to be examined in the course of this project was the influence of having computers with CAS permanently available in the classroom, on the goals, content and organization of teaching mathematics. The influence on pupil and teacher motivation was examined, as well as the possible effects on the curriculum and teaching aids and further education concepts for the teachers. Further information (in German) can be found at:
http://www.acdca.ac.at/projekt/index.htm

CAS III: school year 1999/2000, head of project: H. Heugl (Vienna)

Examination-related and developmental work are to be undertaken with more than 100 test-classes on the following topics: Electronic and Learning Materials (global learning environments in which CAS is used as a tool); TIMSS and CAS supported teaching of mathematics; a manual on the curriculum with special references to CAS; the influence of CAS on examination practice; the new culture of learning with CAS. Further information can be found at:

http://www.acdca.ac.at/english/cas3_99.htm

Whereas the projects CAS I, II and III were (and still are) essentially set up towards concentrating on increasing the use of CAS in high schools, whereby a certain self-dynamics in further developing and widespread use of CAS is expected, the **CAS-Project in Klagenfurt**, which E. Schneider is reporting on here at the CAME meeting, is attempting to gain didactical experiences and research results in using CAS as well as in the cooperation between scientific didactics and teaching practice at commercial high schools. Further information about this project (in German) can be found at:

http://www.uni-klu.ac.at/groups/math/didkctik/arb/casmu.htm

France — by Dominique Guin

1 – Study by the Ministry of Education about evaluation of calculators use in “baccalauréat” (final secondary school examination) and new curriculum.

Email: gras@univ-rennes1.fr

2 – Ministry of Education : research team on CAS (leader A. Hirlimann)

Email : A.Hirlimann@edutel.fr

3 – Paris, university level : distance learning, not only about CAS

Email : jarraud@ ccr.jussieu.fr

http://www.math.jussieu.fr/~jarraud/pcsm.html

4 – Texas Instruments experimentations (30 classes in mathematics/physics)

Email : h-colombat@ti.com

5 – Commission inter-IREM (M.Dupérier) : internet exchanges between secondary teachers integrating DERIVE.

Email : MJ.Duperier@wanadoo.fr

6 – Research project : IREM de Lyon, G. Aldon : CAS use in the context of “Managed Personal Work”, a new concept in the French secondary curriculum. “Problem situations” are to be elaborated, with the aim of developing in students a “scientific practice” similar (in some aspects) to mathematics research practice.

Email : aldon@univ-lyon1.fr

7 – Research project : Paris VII, research team DIDIREM (leader : M.Artigue)

Doctoral Thesis of Badr Defouad: about students’ instrumentation process of the TI 92 from experimentation in two scientific classes (16/17 years old)

Collaboration with J-B Lagrange (Rennes)

Email : Michele.artigue@gauss.math.jussieu.fr ; Lagrange@univ-rennes1.fr

8- – Research project : Montpellier, research team ERES (leader : D.Guin ; L.Trouche)

Comparison between two scientific classes (17/18 years old).

On going research about students’ instrumentation process, and ecological conditions in the classroom in order to get a real integration of the TI 92. Observations of students’ behavior and analysis of their productions through their written reports.

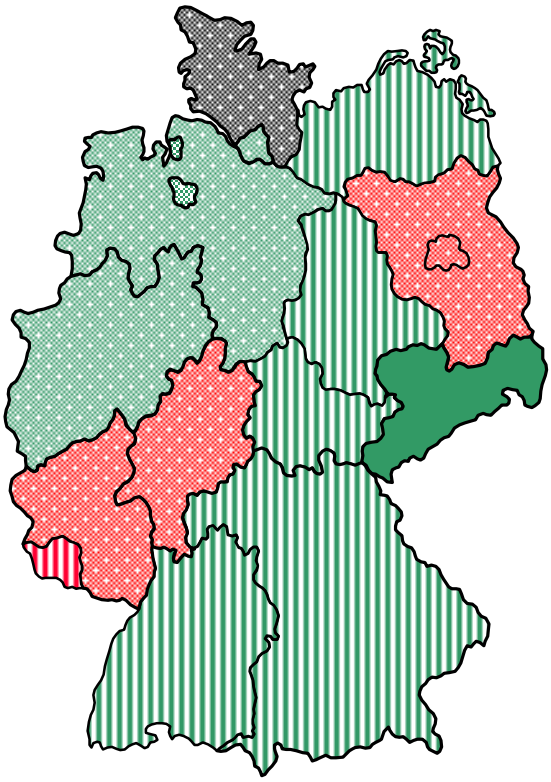
CAS in Germany: some headlines — by Bärbel Barzel

[*NB. The diagram on this page needs to be printed/viewed in colour*]

HISTORY		
Until 1989	**FRG** NO Graphics!	**GDR** Graphics - in research, in class trials
Today	**West-Germany** Mostly CAS, with TI-92 more Graphics in general	**East-Germany** Mostly Graphics With TI-92 more CAS

Class-Trials - an overview

(more than 100 test-classes)



With Class-Trials:

Mecklenburg-Vorpommern: TI-92
Sachsen-Anhalt: Graphics
Thüringen: TI-89
Baden-Württemberg: Maple
Nordrhein-Westfalen: Derive, TI-92
Bremen: TI-92
Hamburg: Derive
Niedersachsen: Graphics, TI-92
Bayern: Graphics (Realschulen)
Sachsen: Graphics,**mandatory in exams**

Without Class-Trials:

(but with a lot of regional activities)
Schleswig-Holstein, Brandenburg, Hessen, Rheinland-Pfalz, Saarland, Berlin

Stripes:

States with central examination (Zentralabitur)

Stars:

States without central examinations (Teachers create the exams)

Curricula - of the single states

- recommendation for the use of CAS in the classrooms and exams

  (when all pupils have access to the same technology) in most of the "Stars-states" (e.g.: NRW, Bayern, BW, Bremen, Hamburg)

- mandatory in the final (central) examinations only in Sachsen: Graphics are mandatory, plan: in 2002 TI-92 instead of GCs, other states will follow (e.g.: Mecklenburg-Vorpommern)

Research Projects

Up to now too little research and implementation of CAS in teacher pre-service-training as well as too little cooperation between universities and teachers who are involved in the class-trials.

Single activities are exemplary, e.g.: University of Giessen (Hans-Georg Weigand), GDM (Society of Didactics of Mathematics)

In-Service-Training:

Two problems:

- Nearly no official training for math-teachers (because of finance!), also none beside the class-trials
- Official training of school administrators are only restricted to the single state.

A chance:

Teachers Teaching with Technology (T^3), a programme for in-service-training, financially based on an agreement between Texas Instruments and University of Muenster offered since 1997 in-service-training for whole Germany.

Israel — by Azriel Levy

I understand that “computer algebra” refers only to symbolic algebraic manipulations by the computer but not the ordinary activities of a graphing calculator, such as drawing graphs and computing numerical derivatives. As for curriculum projects, the only one I know that was conducted in Israeli high schools , is that conducted by Mrs. Hannah Perl of our group [Weizmann Institute]. She started with 2 classes in the fall of 1996. One was taught by Mrs. Perl and the other by another teacher. The other teacher did not measure up to this project. Mrs. Perl is not presenting a talk at the CAME meeting, but can tell privately, or in group discussions, about her experience. The original plan was that the tenth grade class with which she started would study with the TI-92 calculators for 3 years and the pupils would be examined in a special version of the matriculation examination made up for them. At the end of 1997 the Ministry of Education decided not to allow the TI-92's in the matriculation examination, since the bureaucrats decided that any changes in the matriculation examinations will require extra work from them, and the class Mrs. Perl taught switched to TI-82's.

I know also that the group at the Weizmann Institute use Derive in inservice teacher training. Also Michal Yerushalmy of Haifa University has a project of extensive use of computers in high school, but I do not know whether computer algebra is a part of it.

Computer Algebra in Mathematics Education: some news from Italy

Nicolina A. MALARA, Dipartimento di Matematica, Università di Modena e Reggio Emilia, via Campi 21/B, 41100 Modena. E-mail <malara@unimo.it>

The indications that I give concern colleagues working in Departments of Mathematics of Science Faculties, where most of the Italian researches in Maths Education are based. Italy is a country with a long tradition in Geometry, and many studies on innovations with the software "Cabri-Geometre" have been produced, in particular on the issue of proof. In comparison, there have been few studies in "algebraic" fields involving computer algebra. I will simply write down what I know and I apologise to my Italian colleagues for the omission of news about their work. I include addresses so that anyone can directly ask these researchers for more details.

The order of my list is simply geographical.

1. ARZARELLO Ferdinando, Dipartimento di Matematica, Università, via Carlo Alberto 10, 10123 Torino <arzarello@dm.unito.it>. He is involved in various projects (he is the leader of our community of research) which promote the use of various software packages, but in particular the use of Cabri. In this realm he is also studying how dragging modalities can bring students to face the algebraic side of the geometrical questions explored, realizing dynamically a "jeu de cadre" (in the sense of R. Douady) between Euclidean Geometry and algebraic varieties.

BOIERI Paolo, Dipartimento di Matematica, Politecnico, Corso duca degli Abruzzi24, 10129 Torino <boieri@polito.it>; His research is exclusively devoted to the use of multimedia tools in the didactics of maths. In particular he studies the use of Matlab from the beginning of secondary school for exploring the relationship between numerical and symbolic systems. He is also involved in preparing didactical materials for the use of graphical pocket calculators.

2. BOERO Paolo, Dipartimento di Matematica, Università, viaDodecaneso 35, 16146 Genova <boero@dima.unige.it>. He is the leader of many projects and has various collaborations, one of these, with DAPUETO Carlo (<dapueto@dima.unige.it>), concerns mathematical modeling. Various software packages, Derive in particular, are used to bring the students to face differential models and to explore concepts such as continuity, complex numbers, but also for studying equations through the graphical representation of functions.

FURINGHETTI Fulvia, Dipartimento di Matematica,Università, via Dodecaneso 35, 16146 Genova <furinghe@dima.unige.it>. She is involved in studies on the influence of the use of software (including computer algebra) on the conceptions of teachers and students, and on the role of mediation of the computer in constructing concepts and new paths of teaching.

CHIAPPINI Giampaolo, I.M.A.-C.N.R., via de Marini 6, Torre di Francia, 16149 Genova <chiappini@uma.ge.cnr.it>. He is involved in various projects for the study of software in didactical use. He also works in the algebraic realm, in collaboration with F. Arzarello and P. Boero.

3. BAROZZI Giulio Cesare, CIRAM (Centro Interdipartimentale di Ricerca per le Applicazioni della Matematica), via Saragozza 8, 40123 Bologna <barozzi@ciram.ing.unibo.it>. He is involved in the study of various software packages (Mathematica, Maple, Matlab, etc) for the teaching of Calculus both in secondary school and university. He has been devoted to these studies for a long time, and his role in spreading this culture among teachers is well recognized.

4. MARIOTTI Maria Alessandra, Dipartimento di Matematica,Università, via Filippo

Buonarroti 2, 56127 Pisa <mariotti@dm.inipi.it> She is working in a project concerning the study of algebraic equations through various software packages in the biennium of secondary school. She is also involved in some projects with Arzarello and Boero.

5. Maria REGGIANI, Dipartimento di Matematica, Università, via Ferrata 1, 27100 Pavia <reggiani@dimat.unipv.it>. She is working on research on the teaching/learning of algebraic equations from grades 7 to10, and their role for the study of geometrical problems; in this realm she considers also the use of specific software packages for the attribution of meanings to variables and parameters.

5. ACCASCINA Giuseppe, Dipartimento di Metodi e Modelli Matematici per le Scienze Applicate, Università "La Sapienza", viaScarpa 16, 00151 Roma <accascina@dmmm.uniroma1.it>. CANNIZZARO Lucilla, Dipartimento di Matematica, Università "La Sapienza", piazzale A. Moro 5, 00185 Roma <cannizzaro@mat.uniroma1.it>. They study the role of algebraic software in the construction of concepts, in the sense of Tall and Vinner, in particular they study the use of Derive in algebraic geometry, both in secondary school and university; in the first case polynomials and polynomial functions are explored.

6. MORELLI Aldo, Dipartimento di Matematica, Università, via Cintia, complesso universitario monte S. Angelo, 80126 Napoli <morelli@matna2.dma.unina.it>. He has realized a set of didactical units on algebraic equations (of degree greater than two) and a special software package where the classical problems of the limitation, separation and approximation of the real roots of an algebraic equation are investigated. In particular old methods of inquiry and discussion (Descartes, Budan, Fourier, Sturm) are revisited in terms of computational procedures. In his group the use of graphical pocket calculators is also promoted.

7. In my group of research now we are using Derive in grade 8 to visualize algebraic laws (representing families of straight lines or of conics) and to promote in the students: i) the interpretations in graphical terms of the various algebraic elements present in the corresponding formulae (signs, coefficients, operations, letters), ii) the attribution of meaning to variables and parameters, iii) the comparision of "analogous" algebraic formulas (such as $y = 3/x$;$y = x/3$).

Computer Algebra in the Netherlands — by Paul Drijvers

1. Curriculum and assessment at upper secondary level

I confine myself to mathematics education at upper secondary, pre-university level. For university level see the Internet site of Computer Algebra Netherlands (www.riaca.win.tue.nl/can/).

A new curriculum has been developed, with national introduction in August 1999. The final assessment at upper secondary level consists of two parts: a school set assessment and a final national examination that is externally set and internally graded.

2. Technology

The current regulation is that a graphics calculator is required at the final examination, whereas computer algebra is excluded (although the HP38G is allowed as well as the TI-83 Plus, which may contain symbolic features in future). For the school assessment, the authorities recommend the use, or partial use, of a computer, but schools are free to decide for themselves.

3. Research projects

In the fall of 1998, the Freudenthal Institute conducted an explorative case study using the symbolic calculator. This machine turned out to be quite useful in investigative tasks. The sophisticated use of variables and parameters, however, was not always clear to the students. Furthermore, some students were reluctant to use computer algebra for the application of techniques that they had not yet mastered manually. In 1999 a new project was started concerning the role of computer algebra in the learning of mathematics and, more specifically, in the learning of algebraic concepts.

Finally, research will be carried out on the possibilities of computer algebra as a wide-range technological tool. A project that focuses on the use of a computer algebra environment in combination with a text editor (to write mathematical reports) and an Internet browser has been started at the Algemeen Pedagogisch Studiecentrum, an institute for improvement of (mathematics) education

4. Future developments

The Dutch Association of Mathematics Teachers formed an Advisory Board on Computer Algebra and Symbolic Calculator. In May, 1998, this Board concluded that:

- computer algebra should be implemented in upper secondary education;
- research was needed in order to find answers to the pedagogical and curriculum issues that will be raised by this;
- as a computer algebra platform, the PC would be preferred to the symbolic calculator, at least in the short term.

For the full report of the Board (in Dutch!) see: www.euronet.nl/~nvvw/. I expect that this, in combination with the results of current research projects, will lead to an implementation of computer algebra at upper secondary level. I expect, however, that this may take some time, perhaps within the next curriculum revision in about 4 years?

Computer Algebra Related activities in the UK — by John Monaghan

The UK is made up of England, Northern Ireland, Scotland and Wales. The Scottish system is quite different to the others (which are largely similar). Most of the data below relates to E, W & NI. Compulsory schooling is from 5-16. The number of students studying academic maths at 16+ is not high. In E, W & NI it is called A-level (advanced level). A-level Maths contains quite a lot on techniques in calculus of a single variable. A-level examination result determine University entrance. Details of the 5-18 curriculum and assessment may be found on http://www.qca.org.uk/aframe.htm

Ken Ruthven wrote a CAS report for the UK government that contains UK and international findings. The UK aspect reports on a survey of institutional availability, use and value in lessons and student access and ownership. Ruthven, K. (1997) Computer Algebra Systems in Advanced-level Mathematics: a report to SCAA, University of Cambridge School of Education/School Curriculum and Assessment Authority. It may be obtained from Pauline Stephenson, Qualifications and Curriculum Authority, 29 Bolton Street, London W1Y 7PD.

The Scottish Consultative Council on the Curriculum (http://www.sccc.ac.uk/) published a little booklet *Advanced Calculators & Mathematics Education: a paper for discussion and consultation* (1998) which considers the place of CAS type calculators (as well as graphic calculators) in mathematics education.

SCAA set up a number of working groups in the period 1994-96 to assess the potential impact of CAS on the A-level curriculum and examinations. These are not public documents but a summary of the discussions of these working groups is contained in: Monaghan, J. (to appear) Some issues surrounding the use of algebraic calculators in traditional examinations. *International Journal for Mathematics Education in Science and Technology*.

UK journals of interest: *Maths & Stats* - a quarterly newsletter of the Computers in Teaching Initiative. Deals with all aspects of technology at university level. See http://www.bham.ac.uk/ctimath/

Micromath: a quarterly journal aimed at technology in school maths http://mcs.open.ac.uk/cme/micromath/

Both of these journals are primarily ‘professional’ journals and not ‘research’ journals.

CAS work at school and at university level exists, but in discrete pockets. Important work at university level is going on in the METRIC project— http://metric.ma.ic.ac.uk .

The CTM team in Plymouth conducts CAS work at school and university level, runs IJCAME (International Journal of Computer Algebra in Mathematics Education) and the T^3 project. All of these may be reached via http://www.tech.plym.ac.uk/maths/CTMHOME/ctm.html#aims

The T-Time group at Sheffield Hallam University are doing interesting work with a range of technology, especially calculators, that includes CAS work. They focus on engineering maths and real-time data collection. http://www.shu.ac.uk/schools/sci/maths/T-Time/ttime.html

The UK Mathematical Association (http://www.m-a.org.uk/) has published: Oldknow, A, & Flower, J. (1996) *Symbolic Manipulation by Computers and Calculators: Information, Ideas and Implications for mathematics teaching 14 - 21*.

I don't actually know of any 'straight CAS' PhDs, or research projects focused solely on CAS, other than METRIC above, in the UK at the moment. Work which incorporates CAS is going on in Leeds, London and Plymouth.