

rethinking abstraction: designing with digital technologies

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thanks to

Celia Hoyles

- ▲ Windows on mathematical meanings: learning cultures & computers. Kluwer 1996

Philip Kent

- ▲ Metric Project @ Imperial College

preface

- ▲ didactic obstacles
- ▲ integration not 'natural'
- ▲ new complexities
- ▲ computational transposition (specificities and constraints of interface etc.)
- ▲ compatability of knowledge with 'official' mathematics

themes

constructionist design

windows on mathematical abstractions

technologies and cultures

engaging (with) the learner

rethinking abstraction



what are the objects of study?

grain size of objects

transformation of objects determine
interaction

CAS

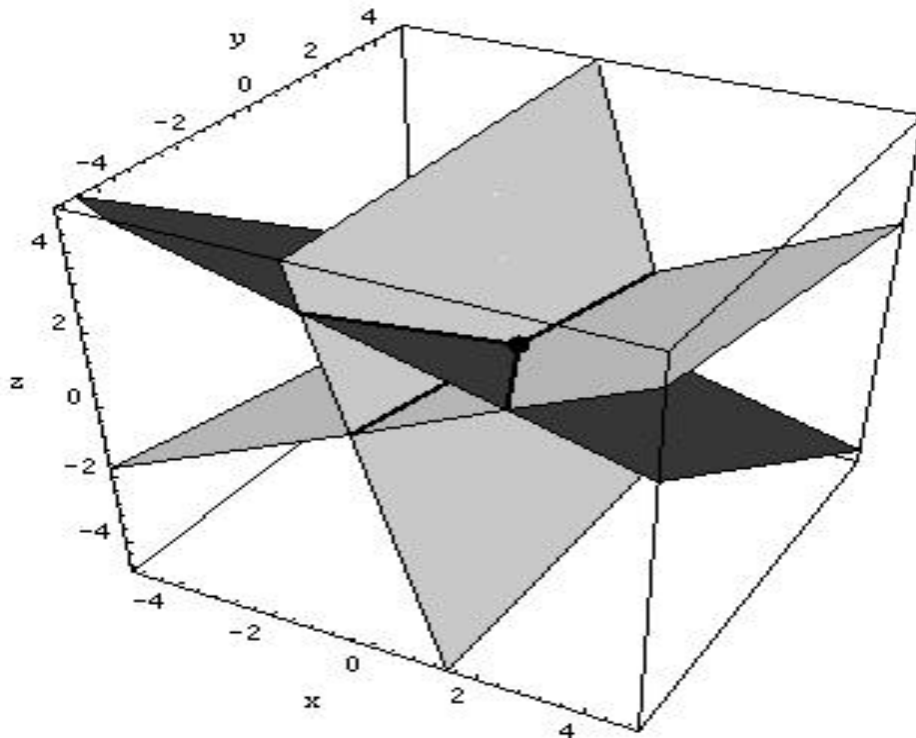
- ▲ strings of symbols
- ▲ tool for manipulation and side effects
- ▲ 20th century technology for 17th century mathematics?
- ▲ if that's what maths is, don't let machines do it!

a first level of abstraction

```
A = {{1,2,3},{4,0,1},{-3,1,5}};  
b = {{1},{1},{1}};  
r = {{x},{y},{z}};
```

```
In[63]:=
```

```
PlotPlanes[A.r==b,{x,-5,5},{y,-5,5},{z,-5,5},  
IntersectQ->True,SolutionQ->True]
```



```
Out[63]=
```

```
{{y -> -(2/35), x -> 6/35, z -> 11/35}}
```

some basic questions

role of 'programming'

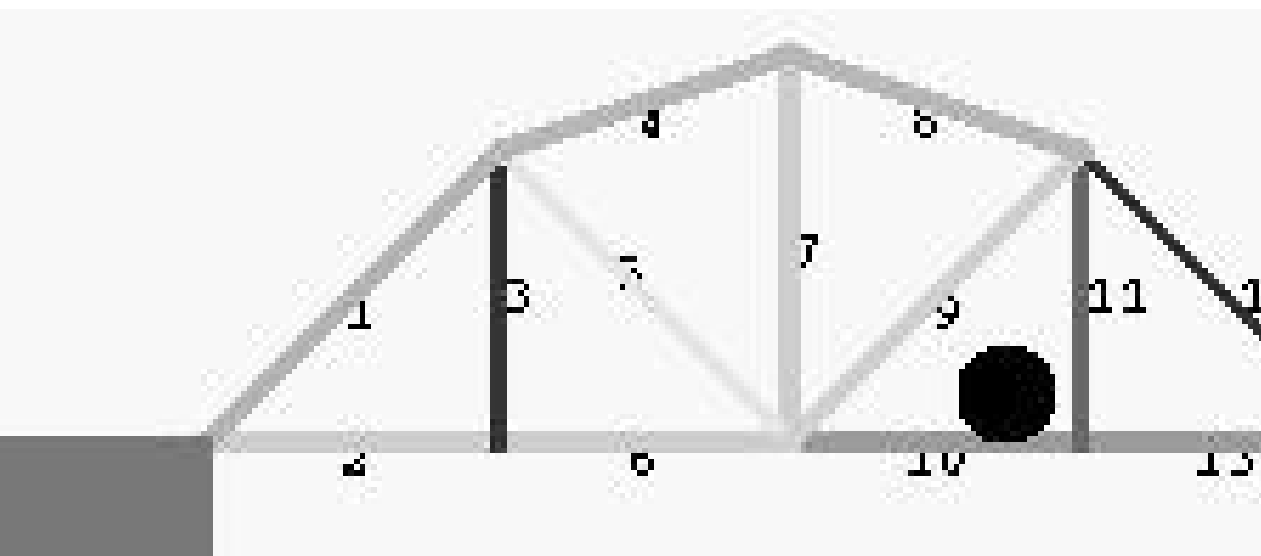
relationship of 'code' to algebra

how black is the black box?

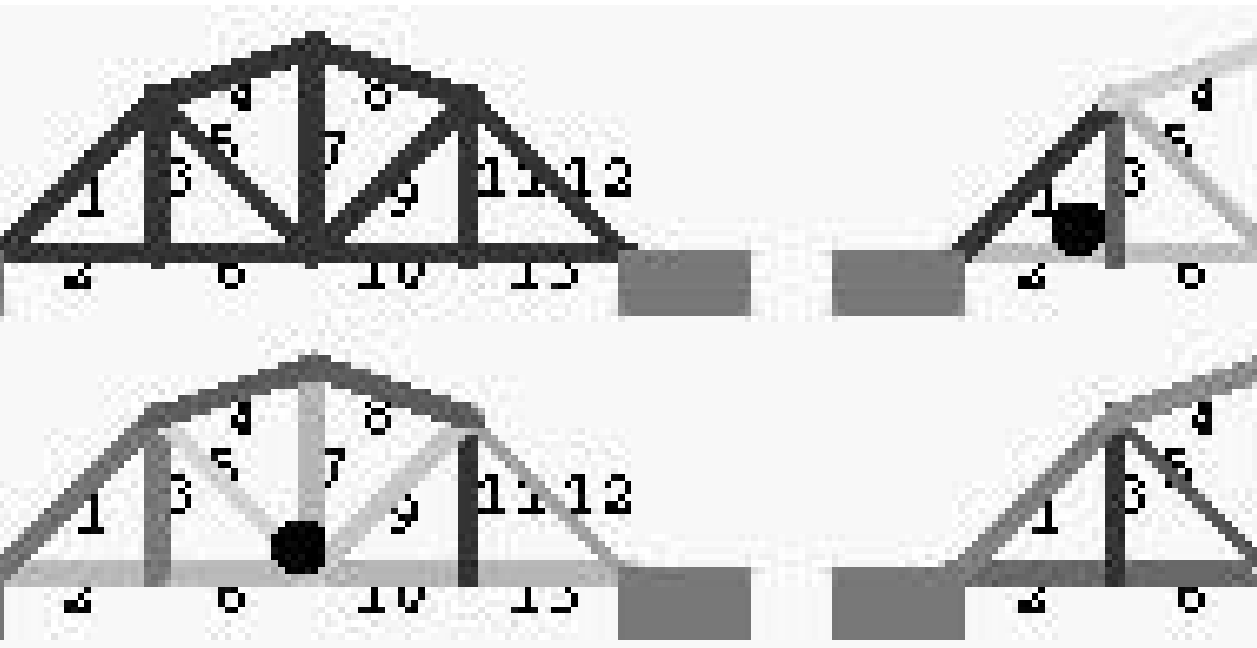
- ▲ transparent tools

- ▲ tuning

rainbow bridge



A colour function maps a numerical force value onto a range of output colours. What function would yield the most useful information about what is going on in the bridge as the test load moves across it? Design a function which would detect the maximum safe load that can cross a bridge given a maximum safe force for any strut.



safe load:

```
loadFun[force_] := If[Abs[force] > 35000,
  DayLevel[1], (*safe load exceeded - output
  e*)
  0.8 * Abs[force] / 35000] (*else output a
  ur spectrum*)
```



the visibility of mathematics

details of process invisible, output visible

interplay between

- ▲ programming (low level)
- ▲ visualisation (high level)

making and criticising representations

- ▲ construction not just multiple representations

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permutations: action & expression

/ cycle [1 2 3 4]

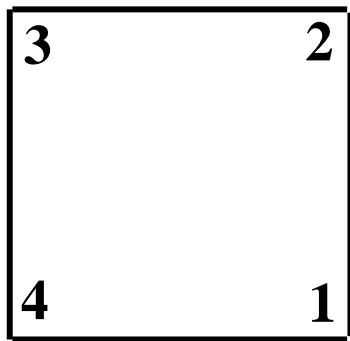
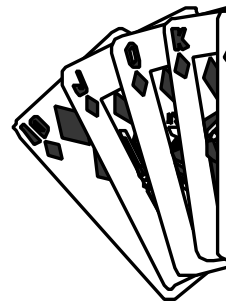
[1]

/ reverse [4 3 2 1]

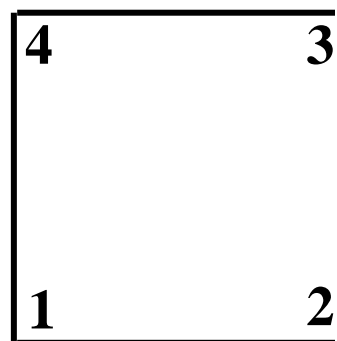
4

/ cycle pack 4

[1]

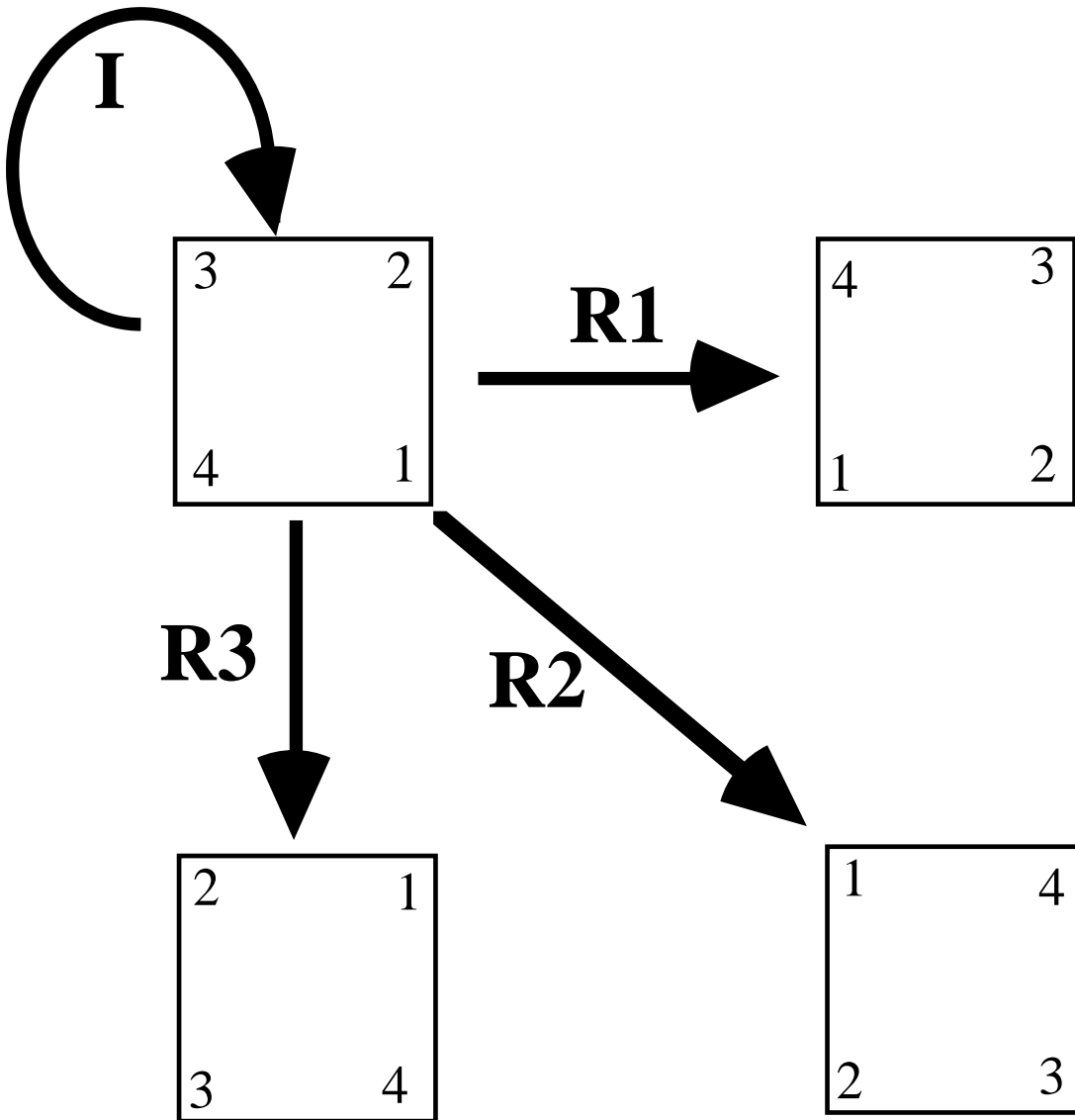


poly1 pack 4



---> poly1 cycle pack 4





The transformations I, R1, R2 and R3

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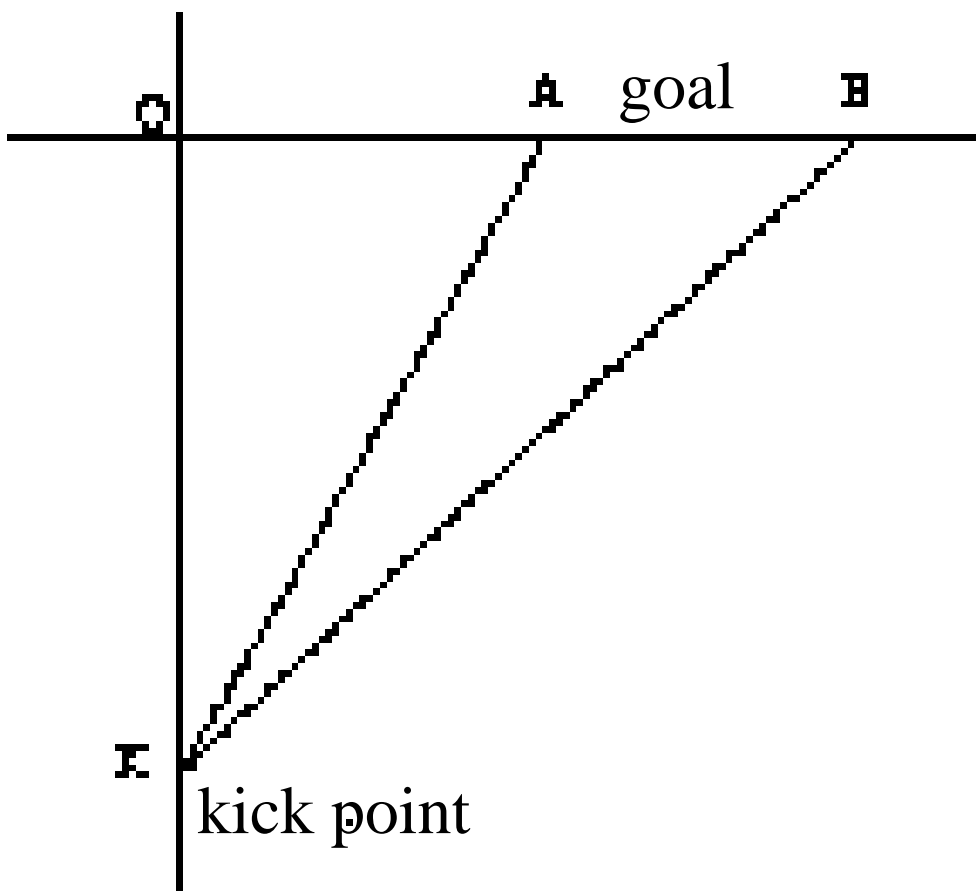
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rethinking abstraction

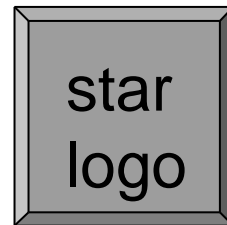
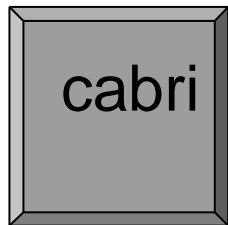
NATO Advanced research workshop (Asilomar, 1993)

- ▲ diSessa A., Hoyles C. & Noss R. (1995)
Computers and Exploratory Learning.
Springer-Verlag

the rugby problem



construction and connection



epistemologies shape cultures (and vice versa)

▲ *Mathematica*

“the world’s only fully integrated technical computing system”

▲ *Maple*

“complete mathematics and visualisation system”

▲ *Mathcad*

“the worldwide standard for technical calculations” ; data-oriented, libraries for engineers; symbolic (Maple) toolbox

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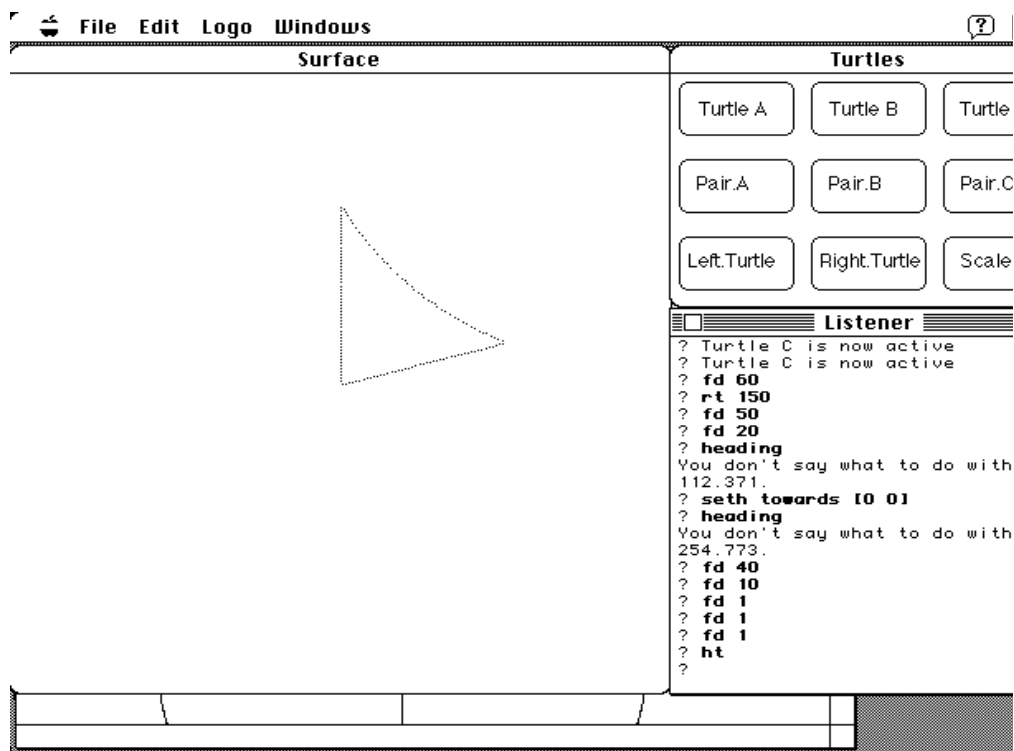
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rethinking abstraction

a non-euclidean world



Stevenson, I & Noss R. (1999) Supporting the evolution of mathematical meanings: the case of non-euclidean geometry. *International Journal of Computers for Mathematical Learning*. 3, 3, 29-254.

learners care about interface

- ▲ phenomenonalising the abstract
- ▲ path as a tool for expressing abstraction
 - straight line = equal steps*
 - turtle needs to walk further to keep up*

more on interface (and grain size)

the playground project

<http://www.ioe.ac.uk/playground>

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abstraction

- ▲ within objects and relationships
- ▲ between objects and learner

revisit technical versus theoretical issue?

situated abstraction

abstracting *within* not *away from* a situation

- ▲ mathematical structures externalised and manipulated within an *appropriate* symbolic or linguistic framework
- ▲ lack the universality of standard mathematical discourse;
- ▲ gain in expressiveness and communicability;
- ▲ webbing internal and external abstraction